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Original Paper

Investigation on propagation mechanism of leakage acoustic waves in horizontal liquid pipelines containing gas bubbles

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A B S T R A C T

Sound speed is essential for leakage detection in liquid pipelines when using acoustic methods, which can be significantly influenced by gas bubbles generated from leakage. The propagation characteristics and mechanism of acoustic waves in horizontal liquid pipelines containing gas bubbles are studied in detail in the present paper. The effect of sound wave frequency, bubble size and bubble distribution pattern on sound speed is studied through numerical simulations. The results show that the acoustic wave generated by leakage of liquid pipelines containing gas bubbles is a multi-frequency signal, and the energy of the signal is mainly concentrated within 200 Hz. In the low-frequency range, the propagation of sound waves has almost no dispersion in bubbly liquid. Sound speed at a certain void fraction is not constant, which is related to the bubble size and distribution pattern. The bubble size affects the gas-liquid heat transfer equilibrium, during which sound speed is affected. For this reason, a thermodynamic correction factor is proposed, which enables the accuracy of the sound speed calculation to reach 98.2%. What's more, sound speed increases non-linearly with the reduction of the bubble distribution space in the pipeline axial direction. This paper establishes a theoretical calculation model of sound speed based on the bubble distribution pattern in the pipeline axial direction, which is in good agreement with the numerical calculation results. The results of this paper provide the basis for applying acoustic leak detection technology in liquid pipelines containing gas bubbles.

1. Introduction

Pipelines are an important transportation means for the world's energy and operation safety is significant (Tu et al., 2023). Effective leak detection and location methods can greatly improve the operation safety of pipelines. Commonly used leakage detection methods for liquid pipelines include the acoustic wave method, intelligent pipe cleaner method, distributed fiber optic method, mass/volume balance method and real-time transient modelling method (Korlapati et al., 2022; Cheng et al., 2023; Yuan et al., 2023). Acoustic leakage detection methods are widely used in liquid pipelines due to high detection accuracy, high sensitivity and low false alarm rate (Hu et al., 2021; Fan et al., 2022; Sitaropoulos et al., 2023). Propagation speed of leakage acoustic waves and the time difference between leakage signals reaching two sensors at both ends of the pipeline are utilized in leakage localization when using acoustic leakage detection methods (Fang et al., 2021; Li et al., 2022). However, when leaks occur in special liquid pipelines (e.g. carbon dioxide dense phase transport pipelines, liquid ammonia transport pipelines, etc.), the fluid near the leakage area changes from single-phase to bubble-like flow due to the pressure change during leakage (Li et al., 2014; Liu et al., 2020; Chen et al., 2024). The sound speed in water is reduced by 50% when the bubble volume content is 0.05%, indicating that even a small number of bubbles in pure liquid media can cause a significant reduction on the sound speed. The presence of air bubbles makes it difficult to apply the acoustic method to liquid pipeline leak detection. Therefore, it is necessary to investigate the acoustic wave propagation mechanism in liquid pipelines containing gas bubbles.

The propagation characteristics of acoustic wave in bubble flow have often been discussed in vertical pipelines in previous studies (see Fig. 1), where bubbles are uniformly distributed in the pipelines. And the results of the studies are mainly applied to downhole

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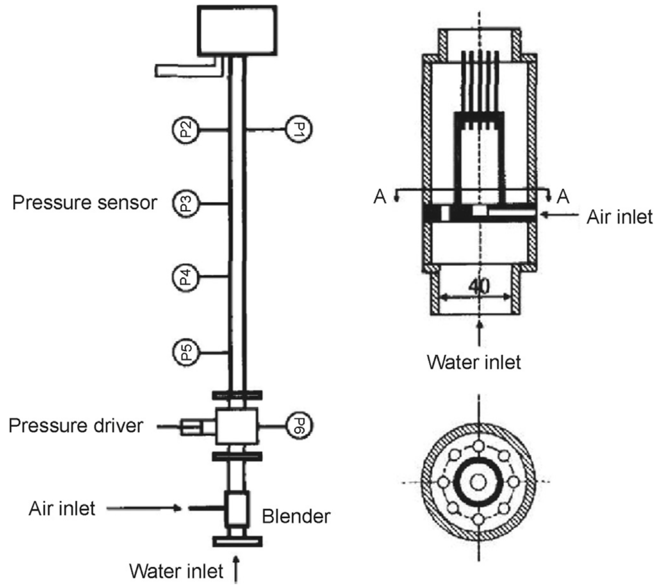


Fig. 1. Experimental apparatus for ideal bubble flow in a vertical pipe by Bai and Huang (2004).

acoustic detection of gas surges (Meng et al., 2015; Wang et al., 2022), mud pulse telemetry (Li et al., 2015, 2022), etc. Costigan and Whalley (1997) investigated the relationship between sound speed and void fraction by experiment. Huang et al. (2004) demonstrated that acoustic waves in bubble flow has dispersive properties, and sound speed is greatly influenced by void fraction and frequency. In contrast, the apparent velocity of the gas and liquid has almost no effect. After which, Bai and Huang (2004) verified the existence of critical frequency for the dispersion phenomenon in bubble flow, above which the acoustic dispersion disappears. Wang et al. (2022) showed that the effect of changes in the local position of bubbles can be neglected at low frequencies. However, in horizontal pipelines, differences in gas-liquid density and viscosity, as well as surface tension, result in bubbles that are mainly concentrated in the upper part of the pipeline (Fig. 2), which

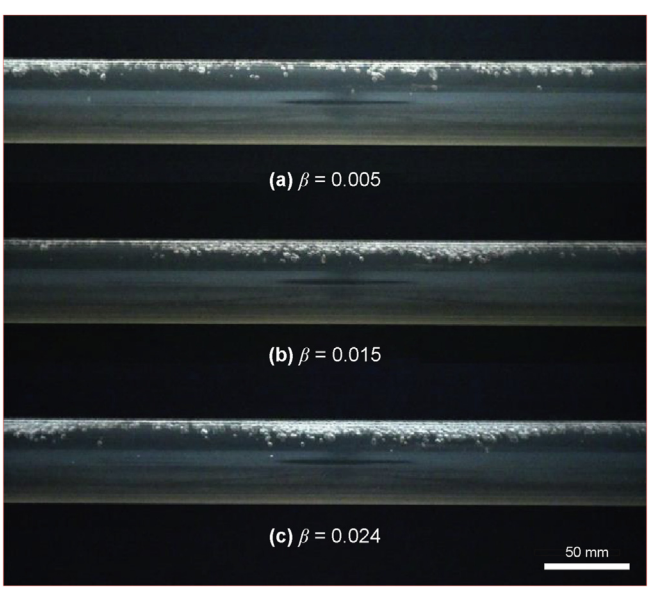


Fig. 2. Distribution of bubbles in a horizontal liquid pipeline.

affects the nature of the leakage fluid and the leakage acoustic propagation medium, but the impact on the acoustic propagation characteristics is unclear. In addition, in Fig. 2, there are differences in bubble size. Moreover, the axial distribution of bubbles generated by leakage pressure perturbations varies due to factors such as the nature of the leaking fluid, the size of the leakage holes, and environmental conditions. And there is a lack of understanding regarding the impact of the non-uniform bubble distribution on the propagation characteristics of acoustic waves in the pipeline. Therefore, the acoustic leakage detection technique needs to consider the variability of bubble distribution in horizontal pipelines and the effect of these differences on the acoustic wave propagation characteristics.

In the theoretical research field, Nguyen et al. (1981) proposed a model for calculating sound velocity in different gas-liquid two-phase flow patterns, based on the phenomenon that sound velocity in a single-phase fluid is affected by the elastic wall. In recent years, the two-fluid model has been widely used to calculate wave velocity in gas-liquid two-phase flow because of its ability to exactly describe the mass, momentum and energy exchange between gas and liquid. Ruggles (1987) combined the two-fluid model with acoustic dispersion relation to obtain the propagation velocity of acoustic waves in bubble-containing liquids. Chung et al. (2004) introduced a new term in the momentum equation, an interfacial pressure jump term based on the surface tension term, to make the two-fluid model hyperbolic. The model accurately predicts the speed of sound, consistent with the findings of Nguyen et al. (1981), although differences emerge between the models, particularly under limiting conditions where Nguyen's model yields unphysical outcomes. Huang et al. (2004) utilized the two-fluid model to develop a mathematical model for pressure wave propagation in bubble-containing liquids, based on the principle of small disturbances and the solvability of the linear equation system. This model's calculated pressure wave velocity was in good agreement with Henry's experimental data. However, the method requires the assignment of an initial value for the numerical solution of the quadratic equation with complex coefficients, which can lead to numerical instability. Consequently, the method has had limited application over the past decade. Xu and Gong (2008) described the pressure source in the gas-liquid momentum equation as a function of the gas-liquid flow velocity, void fraction, its gradient, and differential, thus endowing the two-fluid model with hyperbolic characteristics. By introducing the virtual mass force, a model for the pressure wave velocity in two-phase flows was established. Gubaidullin and Fedorov (2016, 2018) used the two-fluid model to introduce small disturbances, and established the integral-differential equations for the perturbed motion of two-phase mixtures, and obtained an expression for the speed of sound in vapor-air bubble liquids.

The Rayleigh-Plesset equation (Plesset, 1949) and the Keller equation (Keller and Miksis, 1980) are widely used theoretical models for describing the behavior of bubbles, also known as the bubble dynamics model. Researchers used the bubble dynamics model to investigate the oscillation of bubble shapes in incompressible liquids under the influence of acoustic fields (Omoteso et al., 2021; Ni and Pang, 2024). Commander and Prosperetti (1998) established a model for the propagation of small amplitude linear pressure waves in bubbly flow. This model has been adopted by many researchers (Ando et al., 2009; Kargl, 2001), with further modifications such as higher-order scattering of bubbles (Kargl, 2001; Wang et al., 2023), bubble-bubble interactions (Fuster et al., 2014; Zhang et al., 2023), and non-uniform pressure distributions (Zhang and Du, 2015). Based on the model of Commander and Prosperetti (1998), Fuster and Montel (2015) proposed a set of equations to study oscillating vapor-gas bubbles. Zhang et al. (2018,

2022) subsequently improved these equations by considering liquid compressibility and mass transfer. Researchers have established models for calculating the sound speed in bubble flow from different perspectives, and the models are in good agreement with the experimental results of bubble flow in vertical pipelines, but the accuracy is unknown when applied to horizontal pipelines.

Motivated by this open problem, the effects of acoustic wave frequency, bubble size and bubble distribution pattern on the acoustic properties of bubble-containing horizontal liquid pipelines are investigated through numerical simulations, and the influencing mechanism is elucidated, and the sound velocity model applicable to bubble-containing horizontal liquid pipelines is selected and refined according to the simulation results. This study aims to provide theoretical basis and technical guarantee for leakage detection and localization of horizontal liquid pipelines containing bubbles.

2. Modelling of acoustic waves propagating in bubbly fluid

The more commonly used models for calculating the speed of sound in bubble-containing liquids deduced with the two-fluid model are proposed by Xu and Gong (2008) and Gubaidullin and Fedorov (2016). Xu and Gong (2008) described the pressure source of the gas-liquid momentum equation as a function of the gas-liquid flow rate, the void fraction, the void fraction gradient and its differential, making the two-fluid model hyperbolic. The model for the pressure wave velocity in bubble flow was established by introducing the virtual mass force, which is calculated by:

$$c_{\mathrm{t\_X}} = \left\{ \frac{\left(\frac{C_{\mathrm{vm}}\rho_{\mathrm{l}}}{\beta(1-\beta)^2} + \frac{\rho_{\mathrm{g}}}{\beta} + \frac{\rho_{\mathrm{l}}}{1-\beta} \right)}{\left(\frac{\rho_{\mathrm{l}}}{(1-\beta)a_{\mathrm{g}}^2} + \frac{\rho_{\mathrm{g}}}{\beta a_{\mathrm{l}}^2} \right) \left[1 + C_{\mathrm{vm}} \left(\frac{\beta}{1-\beta} + \frac{\rho_{\mathrm{l}}}{\rho_{\mathrm{g}}} \right) \right]} \right\}^{\frac{1}{2}} \tag{1}$$

$$a_{\mathrm{g}} = \sqrt{\frac{\gamma P}{\rho_{\mathrm{g}}}} \tag{2}$$

where c is the sound speed in bubbly liquids, the subscripts t and X represent the two-fluid model and Xu model, respectively. C_{vm} is the virtual mass force coefficient, ρ is the bulk density, a is the sound speed in a single phase, γ is the polytropic exponent, and P is the absolute pressure. β is the void fraction. The subscripts l and g refer to the liquid phase and the gas phase, respectively.

The virtual mass force coefficient C_{vm} can describe the momentum transfer between gas and liquid caused by the relative acceleration motion of two phases at the interface. If the interfacial relative acceleration motion is quite weak, C_{vm} is intended to reach zero, while C_{vm} is heading toward infinity if the interfacial motion is quite intensive. The interaction between gas and liquid in bubble flow is usually intensive, and C_{vm} is taken as infinity. Substituting $C_{\mathrm{vm}} \to \infty$ into Eq. (1), We obtain Eq. (3):

$$c_{\mathrm{t\_X}} = \left[(\beta\rho_{\mathrm{g}} + (1-\beta)\rho_{\mathrm{l}}) \left(\frac{\beta}{\rho_{\mathrm{g}} a_{\mathrm{g}}^2} + \frac{1-\beta}{\rho_{\mathrm{l}} a_{\mathrm{l}}^2} \right) \right]^{-\frac{1}{2}} \tag{3}$$

Gubaidullin and Fedorov (2016) established integro-differential equations for the perturbed motion of two-phase mixtures by introducing small disturbances and obtained a model for speed of sound in bubble-containing liquids, calculated as:

$$\frac{1}{c_{\mathrm{t\_G}}} = \sqrt{\frac{1}{a_{\mathrm{l}}^2} + \frac{\beta(1-\beta)\rho_{\mathrm{l}}}{P}} \tag{4}$$

where the subscript G represents Gubaidullin model.

Commander and Prosperetti (1998) considered the effect of fluid viscosity, heat transfer, and surface tension between gas and liquid on bubbly flow and linearized the acoustic propagation equation with the bubble vibration equation satisfying $(\omega r)/c \ll 1$ to obtain Eq. (5) for sound speed in bubble flow (collectively referred to as the bubble dynamics model in this paper)

$$\frac{1}{c_{\mathrm{b}}} = Re\left(\sqrt{\frac{1}{a_{\mathrm{l}}^2} + \frac{4\pi n r}{\omega_0^2 - \omega^2 + 2b\omega \mathrm{i}}} \right) \tag{5}$$

where the subscript b represents the bubble dynamics model, n is the number of bubbles in a unit volume. ω_0 is the angular resonance frequency for the bubble with the radius of r, ω is the angular excitation frequency, b is the damping coefficient.

The angular resonance frequency for the bubble ω_0 is:

$$\omega_0^2 = \frac{P_0}{\rho_{\mathrm{l}} r^2} \left(Re\Phi - \frac{2\sigma}{rP_0} \right) \tag{6}$$

where, $P_0 = P + 2\sigma/r$ is the undisturbed pressure in the bubble; P is the pressure in the liquid; σ is surface tension coefficient between the liquid and the gas. Φ is a complex function related to the heat conduction, which is defined as:

$$\Phi = \frac{3\gamma}{1 - 3\mathrm{i}\chi(\gamma - 1)\left[(\mathrm{i}/\chi)^{1/2} \coth (\mathrm{i}/\chi)^{1/2} - 1 \right]} \tag{7}$$

where, $\chi = D/(\omega r^2)$; D is the thermal diffusivity of the gas phase.

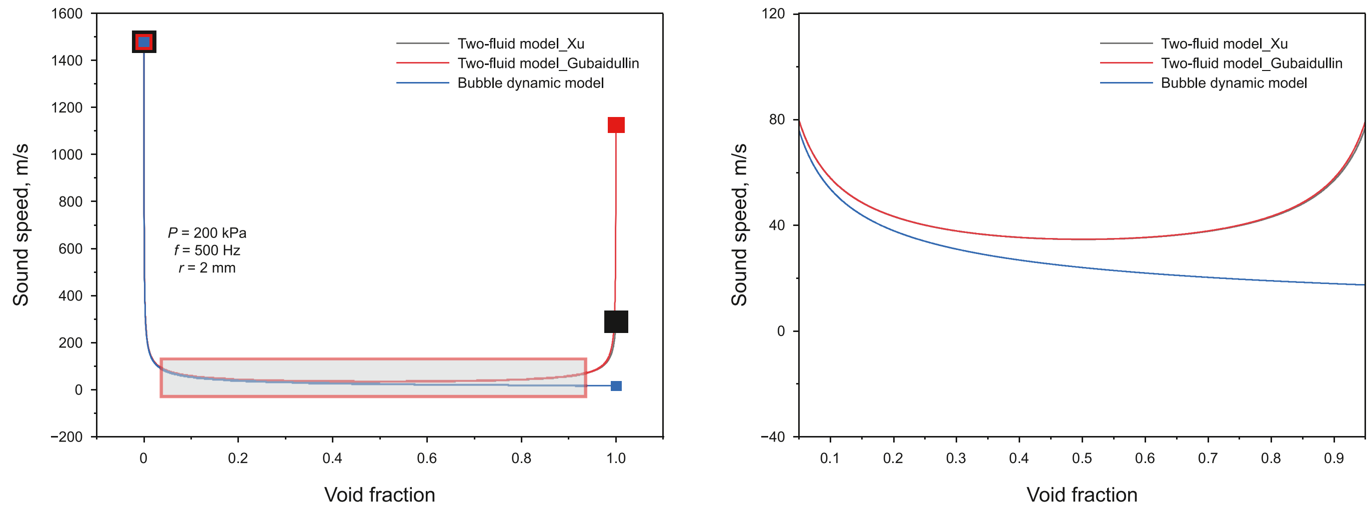
The damping coefficient b is given by:

$$b = \frac{2\mu}{\rho_{\mathrm{l}} r^2} + \frac{P_0}{2\rho_{\mathrm{l}} r^2 \omega} \mathrm{Im}\Phi + \frac{\omega^2 r}{2a_{\mathrm{l}}} \tag{8}$$

where μ is viscosity of the liquid.

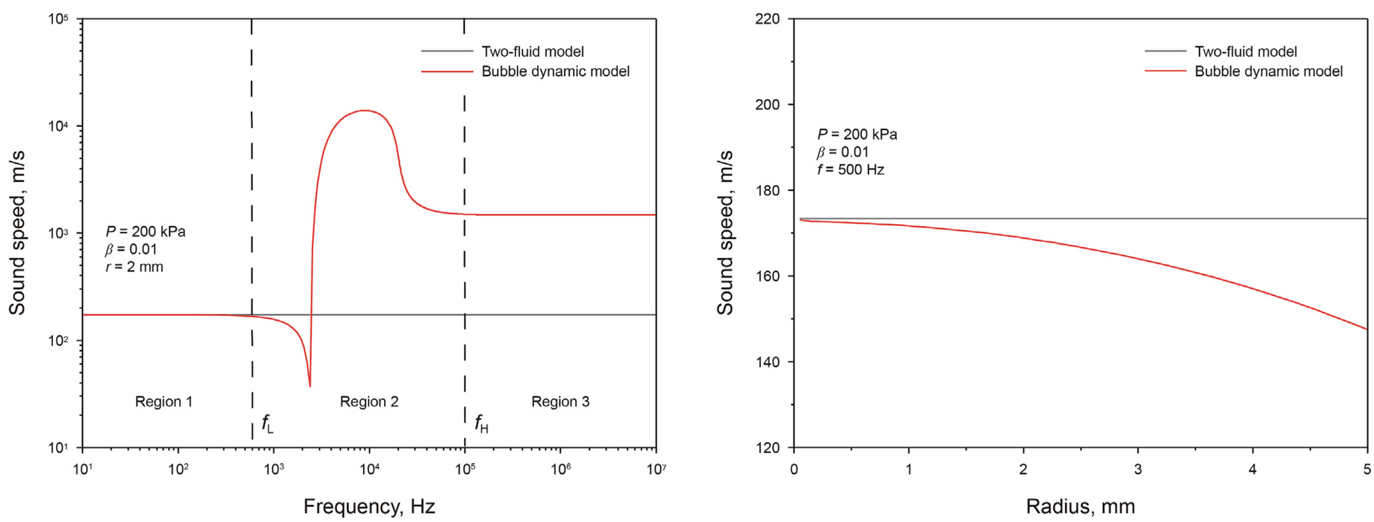
In Eq. (1), a_{g} is an important factor which is depend on the bubble thermodynamic behavior. There is currently controversy over the thermodynamic behavior of bubbles during acoustic wave propagation, with Paillère et al. (2003) and Wang et al. (2022) arguing that the gas inside the bubble is controlled by the isentropic gas equation of state, while Karplus (1961), Brennen (1995), Shamsborhan et al. (2010) and Fu et al. (2020) observed isothermal bubble behavior during low-frequency acoustic wave propagation. The thermodynamic behavior of bubbles in the bubble dynamics model is indirectly reflected by the damping coefficient. Therefore, the thermodynamic behavior of bubbles during acoustic wave propagation will be investigated in this paper.

As can be seen from the model calculation formula, both the Xu model and the Gubaidullin model reveal only the relationship between the sound velocity and the void fraction, while the bubble dynamics model reveals the effects of the void fraction, bubble radius and acoustic frequency on the sound velocity. To compare and analyze the similarities and differences of the models and to quantitatively analyze the characteristics of the effect of each factor on the sound velocity, the calculation results of the models are shown in Figs. 3 and 4. In this section, the thermodynamic behavior of the bubbles is considered as isothermal, i.e., γ is taken as 1.0. Fig. 3 illustrates the effect of void fraction on the speed of sound. It is observed that the results calculated by the Xu and Gubaidullin models are in excellent agreement when $\beta < 0.9$. Furthermore, the



(a) The entire void fraction range **(b)** Partial enlargement

Fig. 3. Effect of void fraction on sound speed.



(a) Effect of frequency **(b)** Effect of bubble radius

Fig. 4. Effect of frequency and bubble radius on sound speed.

results of all three models are very close for $\beta < 0.1$. When β exceeds 0.5, both the Xu and Gubaidullin models show that the sound speed initially increases gradually and then rises sharply with increasing void fraction, and $\beta = 1$, the Xu model reaches the speed of sound in pure gas, which is consistent with the actual situation. However, the Gubaidullin model yields a non-physical result of 1127 m/s at $\beta = 1$. In contrast, the bubble dynamics model shows a gradual decrease in the speed of sound with increasing void fraction, down to 10 $\mathrm{m \cdot s^{-1}}$ at $\beta = 1$, also exhibiting non-physical results.

Based on above analysis, both the Xu model and the Gubaidullin model only describe the relationship between sound velocity and void fraction, and their calculations are consistent when $\beta < 0.9$, but at $\beta = 1$, the Gubaidullin model shows unphysical results. Therefore, only the Xu model results, hereafter referred to as the two-fluid model, will be given in the subsequent comparative analysis of the models at low void fractions, and the Gubaidullin model will not be further discussed. Fig. 4 illustrates the impact of bubble radius and acoustic wave frequency on the speed of sound. It can be found that since the effect of acoustic wave frequency and bubble radius is not considered in the process of the two-fluid model, the sound speed calculated by the two models is more consistent only in the low frequency range and with small bubble sizes. In addition, the bubble dynamics model shows that the sound velocity in bubble-containing liquids can be divided into three regions according to the two critical frequencies (f_L and f_H), and for the calculation of the low critical frequency f_L and the high critical frequency f_H, referring to Zhang et al. (2018). In region 1 and region 3, the effect of frequency on the speed of sound is negligible, and since the acoustic wave frequency in region 3 is much higher than the resonance frequency of the bubble, the presence of the bubble does not have a significant effect on the sound speed in the original medium, so the constant is approximately equal to the sound speed in the pure liquid phase. In region 2, the acoustic frequency is between the low-frequency and high-frequency limits, the acoustic propagation speed is closely related to the frequency, and the sound speed reaches the minimum when the acoustic frequency is close to the bubble resonance frequency. In addition, the bubble dynamics model's results indicate that the sound speed decreases as the bubble radius increases at $f = 500$ Hz.

In summary, the bubble dynamics model clearly describes the effects of void fraction, bubble size and acoustic frequency on sound velocity. However, it involves solving a multitude of complex equations, complicating the calculation process. Conversely, the Xu and Gubaidullin models illustrate the relationship between void fraction and sound speed, and the computational equations are simpler, but fail to account for the effects of bubble size and acoustic frequency. Worth mentioning is that the results of three models are similar in the low-frequency range and for small bubble sizes. And for $\beta < 0.9$, the Xu and Gubaidullin models agree, but at $\beta = 1$, both the bubble dynamics and Gubaidullin models produce non-physical results.

3. Numerical simulation model

3.1. Control equations

The propagation of acoustic waves in a horizontal liquid pipeline containing gas bubbles was simulated using Ansys Fluent and the VOF model was chosen (Xue et al., 2023). The governing equations including mass, momentum and energy equations can be formulated as:

$$\frac{1}{\rho_k}\left[\frac{\partial}{\partial t}(\beta_k\rho_k) + \nabla\cdot(\beta_k\rho_k\mathbf{u})\right] = 0 \tag{9}$$

$$\frac{\partial}{\partial t}(\rho\mathbf{u}) + \nabla\cdot(\rho\mathbf{u}\mathbf{u}) = -\nabla P + \nabla\cdot\tau_{\text{eff}} + \mathbf{F}_\sigma \tag{10}$$

$$\frac{\partial}{\partial t}(\rho E) + \nabla\cdot[\mathbf{u}(\rho E + P)] = \nabla\cdot\left(k_{\text{eff}}\nabla T + \tau_{\text{eff}}\cdot\mathbf{u}\right) \tag{11}$$

$$\sum_{k=1}^{n}\beta_k = 1 \tag{12}$$

where $\mathbf{u}$ is the velocity vector, τ_{eff} is the stress tensor, $\mathbf{F}_\sigma$ is the surface tension vector of gas-liquid, E is the energy, k_{eff} is the effective thermal conductive coefficient, T is the temperature. The subscript k refers to the phase of k.

3.2. Numerical simulation methods

3.2.1. Physical model

The pipeline model is shown in Fig. 5, with a length of 6 m and diameter of 40 mm. The inlet condition was for wall condition and the outlet condition was for pressure outlet. And the outlet pressure was controlled by Eq. (13). To avoid the effect of reflections from the inlet wall on the results, a margin of 3 m is left at the pipeline inlet, and the actual simulation length is 3 m. Therefore, the signal is transmitted at the pipeline outlet and received 3 m distance from the outlet. The bubble size and its spatial distribution are defined using the DEFINE_INIT macro.

The simulated acoustic wave was specified by User Defined Function (UDF), with the procedure detailed in the Appendix. The control equation for frequency f is shown below:

$$p = \begin{cases} 200 + 40\sin(2\pi ft) & t < 1/f \\ 200 & t \ge 1/f \end{cases} \tag{13}$$

where p is the gage pressure in kPa, t is the time in s.

3.2.2. Simulation of acoustic waves propagating in liquid pipelines with bubbles

Firstly, to verify the reliability of the simulation, the propagation of acoustic waves at constant frequency ($f = 500$ Hz) was simulated in liquid pipelines with bubbles global distribution, where the bubble radius is 2 mm and the void fractions are 0.0025–0.10.

In section 2, the time-frequency characteristics of the bubble flow leakage acoustic wave were firstly shown. Then, the effect of acoustic frequency on sound speed was explored in the low-frequency range ($f \ll 2\pi\omega_0$), where the bubble radius is 2 mm and the void fraction is 0.01.

To investigate the thermodynamic behavior of bubbles during acoustic propagation, acoustic propagation for the global distribution of bubbles was simulated in a wide range of parameters with void fraction at 0.0025–0.05 and bubble radius at 0.5–5 mm in section 3.

Finally, combining the scenario of bubble distribution in horizontal liquid pipelines, the effects of the bubble distribution pattern on the propagation of sound wave was simulated, in which bubbles distributed locally in the pipeline axial direction and pipeline cross-section, where the void fractions are 0.0025–0.05, the bubble radius is 2 mm, and the wave frequency is 500 Hz. The details are shown in Table 1 and the corresponding bubble distribution is shown in Figs. 6 and 7.

3.2.3. Data processing methods

Fig. 8 shows typical results of the simulation ($\beta = 0.01$, $r = 2$ mm, $f = 500$ Hz). The pressure field at different times reflects the propagation of the acoustic waves, with isobaric conditions in pipeline cross-sections, thus illustrating that the acoustic waves propagate as plane waves in bubbly fluid. By extracting the pressure at the monitoring point (shown in Fig. 5) as a function of time, the peaks in the excitation and reception signals are chosen as feature points to determine the time delay. In addition, the waveform of the received signal changes significantly compared to the excitation signal (see Fig. 5), indicating a non-linear change during the wave propagation process. The reason is that the acoustic wave amplitude in this paper belongs to finite amplitude, the wave velocity c at each point of the disturbance is related to the magnitude of the mass velocity u at each point. While in small amplitude waves, the wave velocity c at any point is equal, thus the waveform does not change. If the wave velocity at the crest is $c + (n+1)u/2$, n is a constant determined by the liquid itself, the wave velocity at the trough is $c - (n+1)u/2$. The speed of small amplitude waves in the same conditions can be obtained as the average of the peak and trough speed of finite amplitude waves. In this paper, the wave velocity of small amplitude waves is studied, and the propagation of finite amplitude waves will be discussed in the future. Therefore, in this paper, the simulated sound speed is calculated as:

$$c_s = \frac{L/\Delta t + L/\Delta t'}{2} \tag{14}$$

where L is the calculated pipeline length, that is 3 m. $\Delta t = t_2 - t_1$ is the wave crest delay time, $\Delta t' = t_2' - t_1'$ is the wave trough delay time. The subscript s represents numerical simulations.

4. Results and discussion

4.1. Model verification

To verify the accuracy of the numerical model, a series of simulations with varying void fractions at a small bubble size and low frequency were performed. The bubbles are globally distributed throughout the calculation domain, where the void fractions are

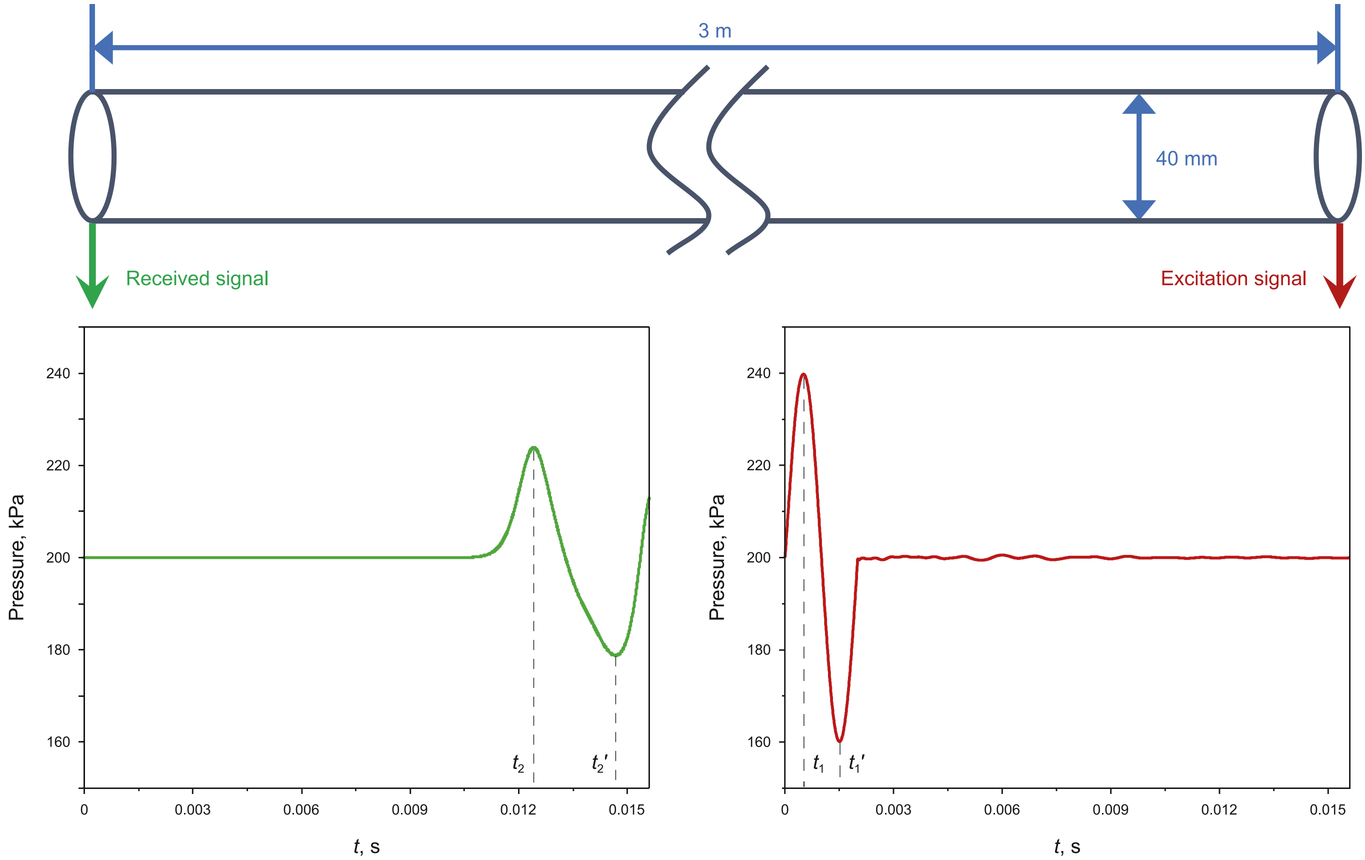


Fig. 5. Numerical simulation model of acoustic wave propagation in bubbly liquid pipelines.

Table 1
Simulation of acoustic wave propagation in a liquid pipeline with non-uniform bubble distribution.

Category	Proportion of space containing bubbles
Non-uniform distribution of bubbles in the pipeline cross-section	0.25 0.5 0.75 1
Non-uniform distribution of bubbles in the pipeline axial direction	0.33 0.5 0.67 0.83 1

Comments: $\beta = 0.0025{-}0.05$, $r = 2$ mm, $f = 500$ Hz.

0.0025–0.10. Fig. 9 shows the simulated results compared to the theoretical results.

As can be seen in Fig. 9, the simulation results agree well with the two-fluid model calculations when the polytropic exponent is taken as 1.0, with a maximum error of 2.8%. The errors mainly come from reasonable assumptions in the derivation of the theoretical model, numerical calculation methods, etc. Despite the existence of errors, they are within acceptable limits and do not materially affect the conclusions of this study. Thus, the isothermal two-fluid model can better describe the propagation behavior of acoustic waves in bubbly liquid at low frequencies, while verifying the accuracy of the numerical simulation method.

4.2. The effect of acoustic frequency

The dynamic pressure sensor (106 B) was used to collect the bubble flow pipeline leakage signal, and the signal was analyzed in the time and frequency domains using a short-time Fourier transform. The time-frequency characteristics of the acoustic wave of pipeline leakage at different void fraction and pressures are shown in Figs. 10 and 11, respectively. Here, the leakage hole is circular with a diameter of 6 mm and it is located directly beneath the pipeline. As can be seen from Fig. 10, the leakage sound wave is a multi-frequency signal and the signal energy is mainly concentrated within 200 Hz. The frequency distributions of leakage signal are similar at varying void fraction. According to Fig. 11, the peak value of the leakage signal increases with the increase of pressure in the pipeline, but the signal energy is still concentrated within 200 Hz.

Since the leakage signal is consisted of signals with different frequencies and the energy is mainly concentrated in the low frequency range (< 200 Hz). Therefore, in the low-frequency range,

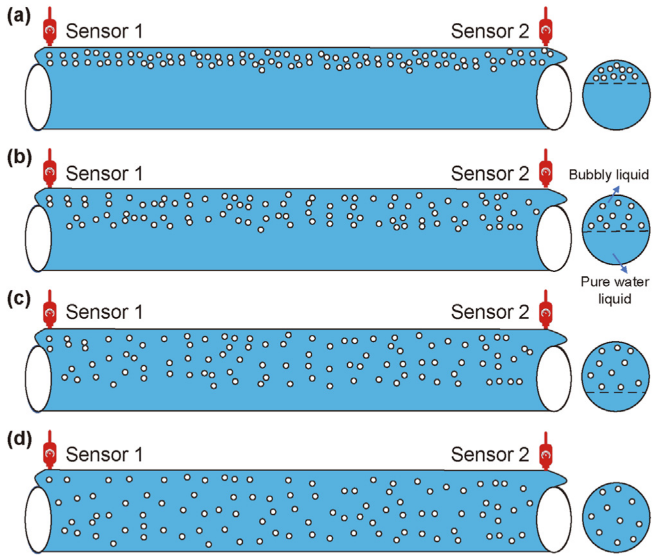


Fig. 6. Bubble distribution categories in pipeline cross-section.

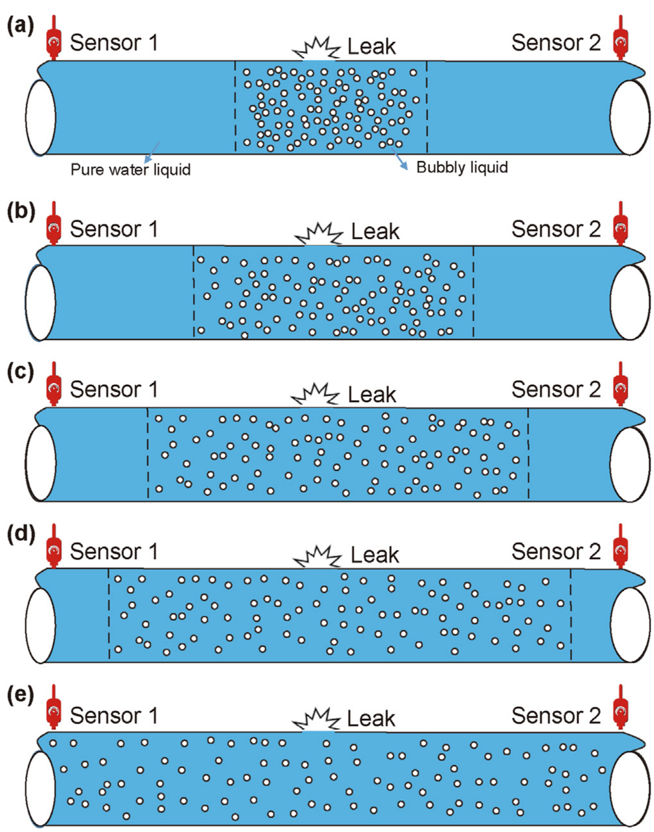


Fig. 7. Bubble distribution categories along the pipeline axial direction.

numerical simulations were carried out with varying frequencies ($f = 100$, 200 and 500 Hz) and constant bubble radius ($r = 2$ mm) at void fraction $\beta = 0.01$. Fig. 12 shows the simulated and calculated results of two theoretical models at the same conditions. The results show that the sound speed is negligibly affected by the acoustic frequency at low-frequency range. And the relative error is less than 1% between the simulated results and two theoretical results. It can be concluded that there is almost no dispersion at low-frequency and that the two-fluid model is still highly accurate in investigating sound speed at low frequency.

4.3. The effect of bubble radius

4.3.1. Influencing mechanism

Fig. 2 shows that the size of bubbles in the bubble flow pipeline is highly dispersed. Therefore, to investigate the effect of bubble size on the propagation velocity of acoustic waves in bubbly liquid, models with different void fractions ($\beta = 0.0025$, 0.005, 0.01, 0.03 and 0.05) and bubble radius ($r = 0.5$–5 mm) were carried out at the frequency of 500 Hz. Fig. 13 shows the simulated results and two theoretical models calculation results at the same conditions. The simulation results show that at a constant void fraction, the sound speed first remains almost constant and then gradually increases as the bubble radius increases. When $r > 3$ mm, the deviation between the simulated and the theoretical results are not negligible, especially at $\beta = 0.0025$ and $r = 5$ mm, where the deviation from the two-fluid model is 8% and from the bubble dynamics model is 26%.

The reason for the deviation of the bubble dynamics model is that the assumption for the model $(\omega r)/c \ll 1$ is no longer satisfied when $r > 3$ mm. Therefore, the bubble dynamics model is only applicable in bubbly liquid with low void fractions and small bubbles to investigate propagation characteristics of acoustic waves, as reported by Brennen (1995), Commander and Prosperetti (1998), Wang et al. (2022) and Zhang et al. (2018). In summary, the two-fluid model is advantageous for calculating the propagation velocity of low-frequency acoustic waves. However, there are still large deviations at large bubble sizes, and influencing mechanisms of bubble sizes on the sound speed will be discussed in detail afterwards.

Fig. 14 shows the temperature inside the bubbles and surrounding liquid phase temperature at different bubble sizes when the acoustic wave crest passes through. When $r < 3$ mm, the temperature inside the bubbles is equal to the temperature of the surrounding medium, which illustrates that the gas-liquid phase is always in thermal equilibrium. That is, the acoustic wave propagation process is isothermal. Calculating gas phase sound speed by Eq. (2), the propagation of acoustic wave is firstly considered to be an isothermal process, so γ is taken as 1.0. Therefore, when $r < 3$ mm, the maximum deviation of the two-fluid model calculation results compared to the simulated results was low, with the deviation of approximately 1.47%. However, when r are 4 mm and 5 mm, the maximum deviation is 4% and 8% respectively, which is due to the increase of the bubble size, the heat of the temperature rise inside the bubble caused by the acoustic wave crest is not dissipated in time, thus leading to the existence of temperature difference between the gas and liquid phases, so that the pipeline shows a non-thermal equilibrium state. At this point, the acoustic propagation thermodynamic process deviates from the isothermal process. If γ is taken as 1 to calculated the acoustic wave propagation velocity, it will seriously underestimate the actual speed of sound.

4.3.2. Proposal and validation of thermodynamic correction coefficients

Considering the differences in the thermodynamic processes of acoustic wave propagation at different bubble sizes, γ in the two-fluid model is corrected based on the simulated speed of sound at different bubble sizes with $\beta = 0.0025$ and this paper proposes a thermodynamic correction factor f_T as:

$$\gamma' = f_T \cdot \gamma \tag{15}$$

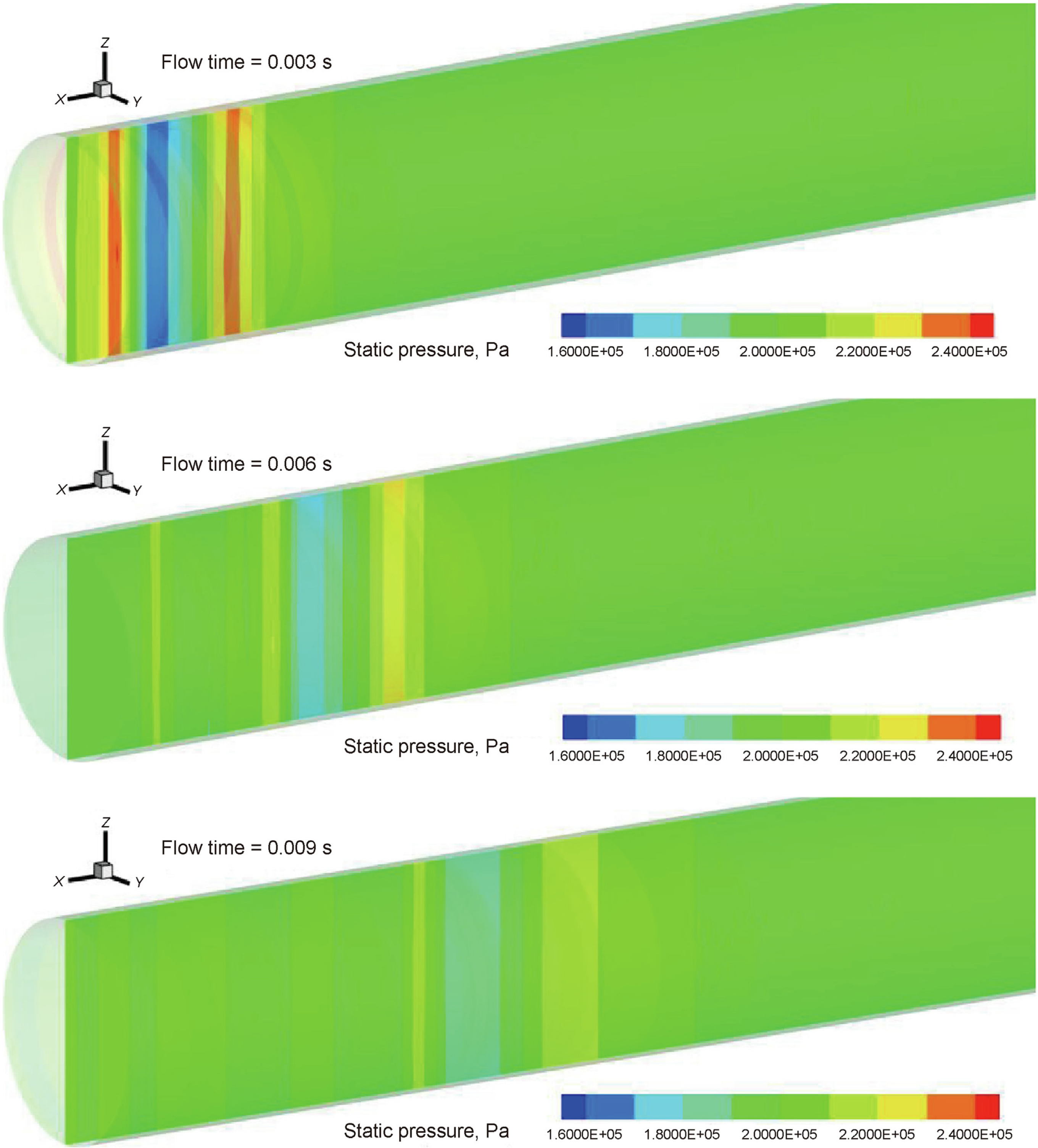


Fig. 8. Acoustic wave propagation in a bubbly fluid pipeline.

$$f_{\mathrm{T}} = \frac{1.005 - 152.153r}{1 - 142.816r - 2812.326r^2} \tag{16}$$

Fig. 15 shows the sound speed after considering f_{T} at different bubble sizes with different void fractions. For $\beta = 0.0025$, the maximum deviation between the two-fluid model calculations considering f_{T} and simulated results is reduced from 8% to 1.8%. It can also be seen from Fig. 15 that f_{T} applied to other void fractions improves the calculation accuracy of the two-fluid model. Therefore, the proposal of f_{T} in this study is universally applicable.

4.4. The effect of bubble distribution patterns

In horizontal liquid pipelines, bubbles are mainly concentrated

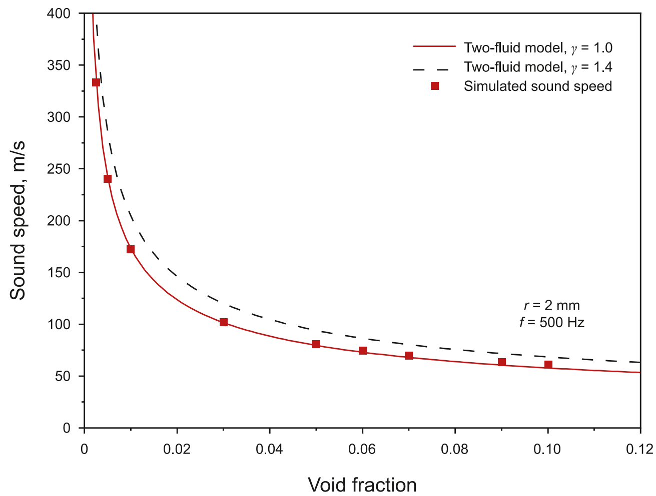


Fig. 9. Simulated and theoretical sound speed.

in the upper part of the pipeline. Additionally, the distribution range of gaseous bubbles along the pipeline axis varies due to differences in the nature of the leakage fluid, the size of the leakage point, and environmental conditions. The effect of the bubble distribution pattern in the pipeline cross-section and axial direction on the sound speed between neighboring sensors (collectively referred to as the overall sound speed V in this paper) was investigated when other variables are constant.

4.4.1. Non-uniform distribution of bubbles at the axial direction of the pipeline

The effects of the spatial size of the bubble distribution in the pipeline axis on the overall sound speed V was investigated at $\beta = 0.0025$–0.05 (see Fig. 7). The simulation results are shown in Fig. 16. To ensure the generality of the results, the independent variable is defined as the proportion of bubble distribution space to the simulation domain. As shown in Fig. 16, the speed of sound is minimized when bubbles are globally distributed at a certain void fraction, and the sound speed increases non-linearly as the spatial proportion of the bubble distribution decreases.

Fig. 17 shows the propagation of an acoustic wave through a liquid pipeline containing bubbles which are not uniformly distributed in the axial direction. Since the acoustic wave propagates as a plane wave, the propagating medium is not always the same along the direction of wave propagation.

When bubbles are distributed in only a portion of the pipeline, two types of liquids (the bubbly liquid and pure water liquid) exist,

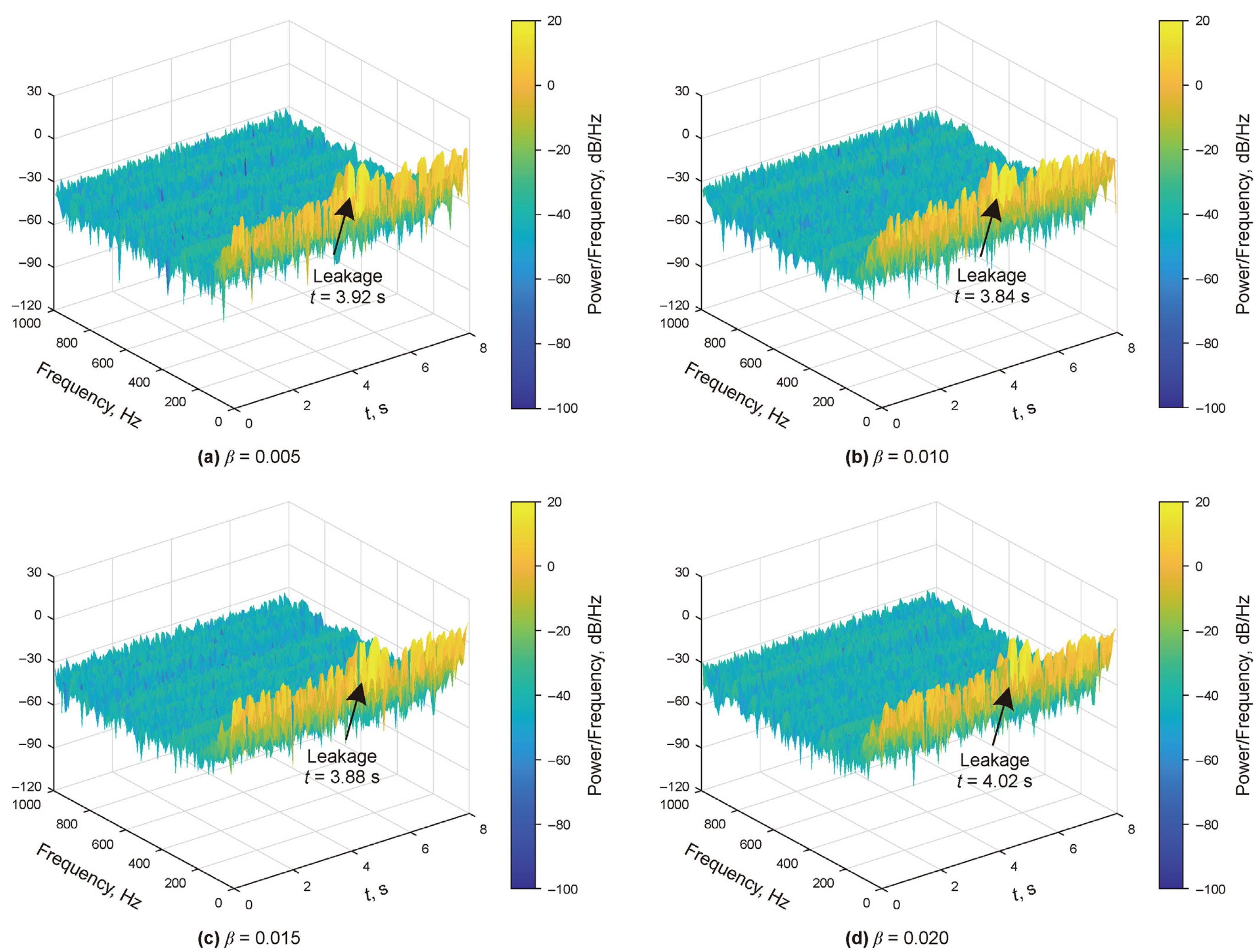


Fig. 10. Time-frequency characteristics of bubble flow pipelines leakage acoustic wave at $p = 0.2$ MPa and $\beta = 0.005$–0.02.

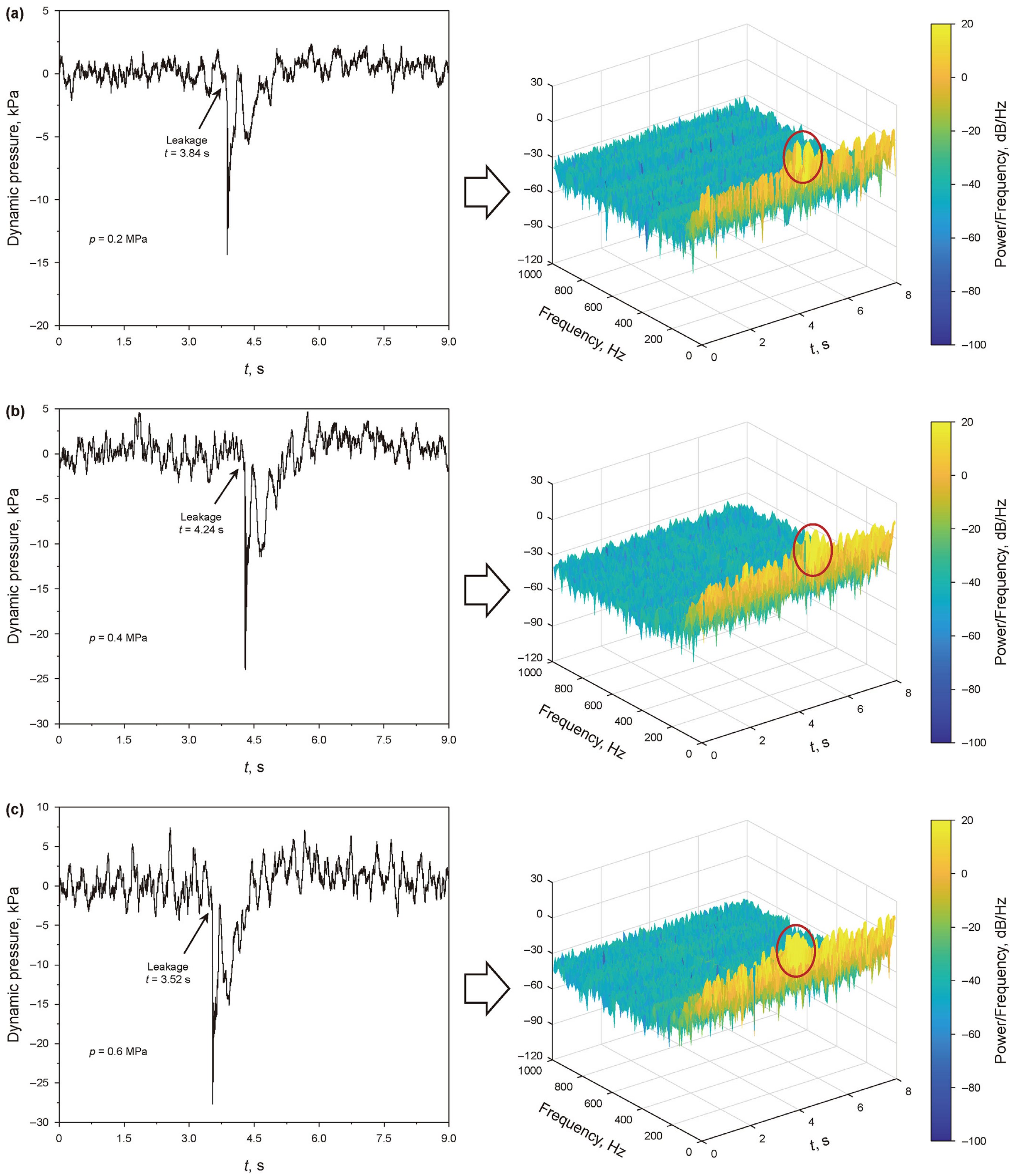


Fig. 11. Time-frequency characteristics of bubble flow pipelines leakage acoustic wave at $\beta = 0.01$ and $p = 0.2–0.6$ MPa.

as shown in Fig. 17. An interface exists between the two types of liquids, where sound waves are reflected due to the difference in the characteristic impedance of the media on either side of the interface (Xu, 2003). The characteristic impedance of the medium Z is calculated as:

$$Z = \rho c \tag{17}$$

where $\rho = \rho_l(1 - \beta) + \rho_g \beta$ is density in bubbly liquids.

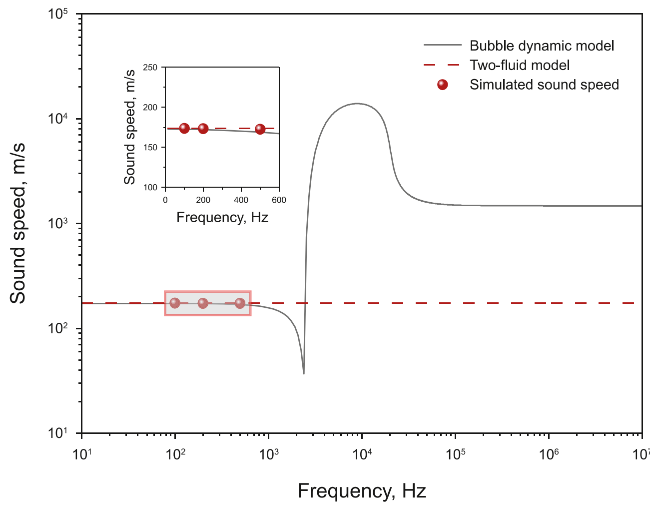


Fig. 12. The variation of sound speed versus frequency.

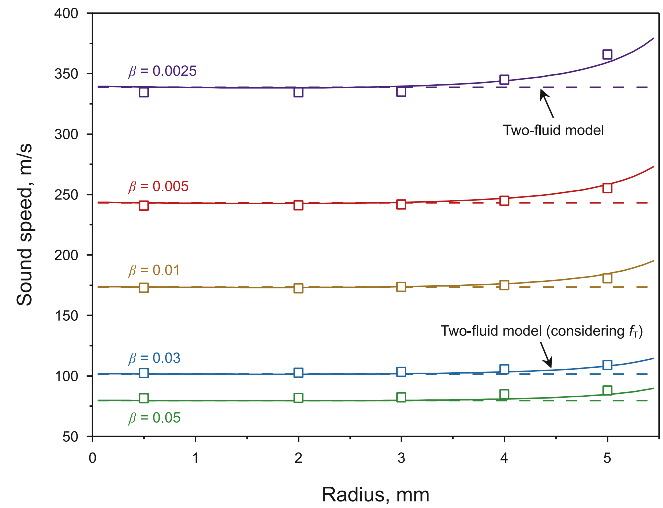


Fig. 15. Sound speed of using the thermodynamic correction factor f_T.

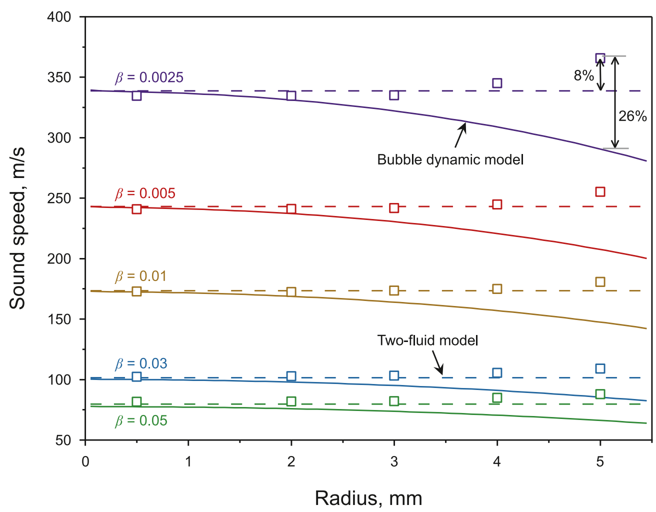


Fig. 13. Sound speed at different void fraction and bubble radius.

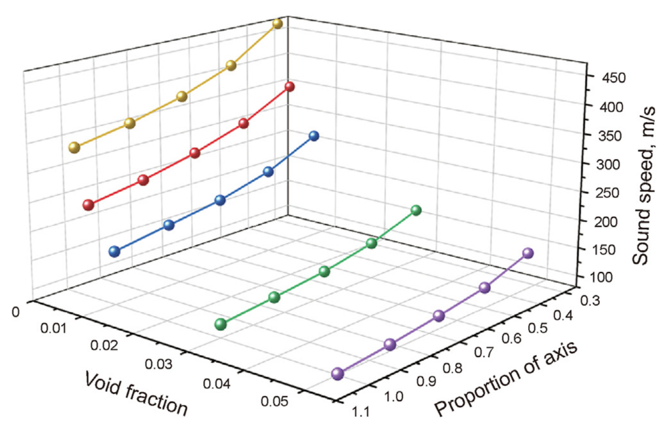


Fig. 16. Overall sound speed versus proportion of bubble axial distribution space.

For example, the void fraction is 0.05 and the spatial proportion of bubble axial distribution is 0.333 (see Fig. 17). The characteristic impedance is 4.39×10^4 Pa•s/m for bubbly liquid and 1.48×10^6 Pa•s/m for pure water liquid. Although a 34-fold difference in the characteristic impedance of adjacent liquid media directly affects the sound pressure distribution at the interface (Xu, 2003), whether there is an elastic hindrance to the speed of sound wave propagation requires further study.

In Eqs. (1), (4) and (5), the void fraction and the sound speed are in one-to-one correspondence. At a constant void fraction, the sound speed can be calculated by Eqs. (1), (4) and (5) when the bubbles are globally distributed in the pipeline, but Eqs. (1), (4) and (5) fail when there are two liquids in the pipeline (see

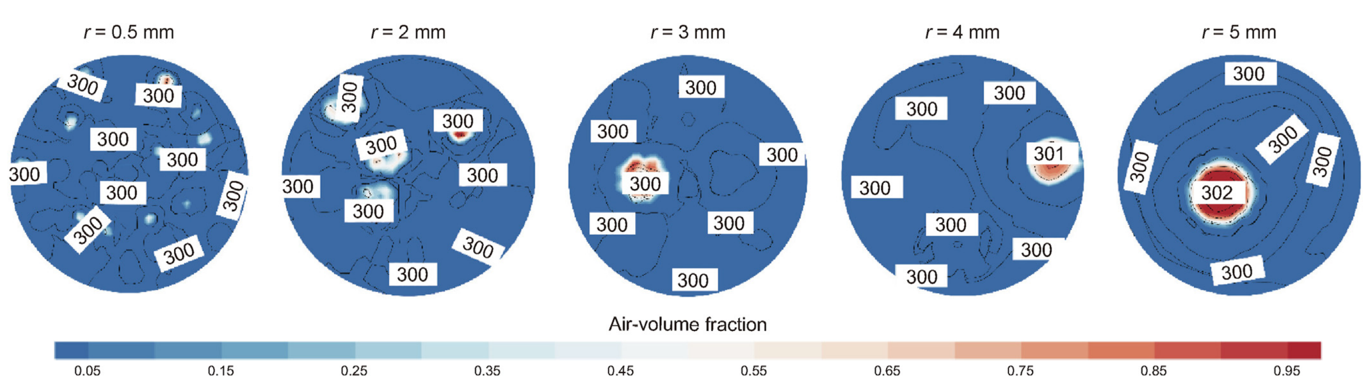


Fig. 14. The effect of bubble sizes on the temperature difference between the gas and liquid phases. $\beta = 0.01$, $f = 500$ Hz. The lines are temperature contours and the numbers indicate temperatures in K.

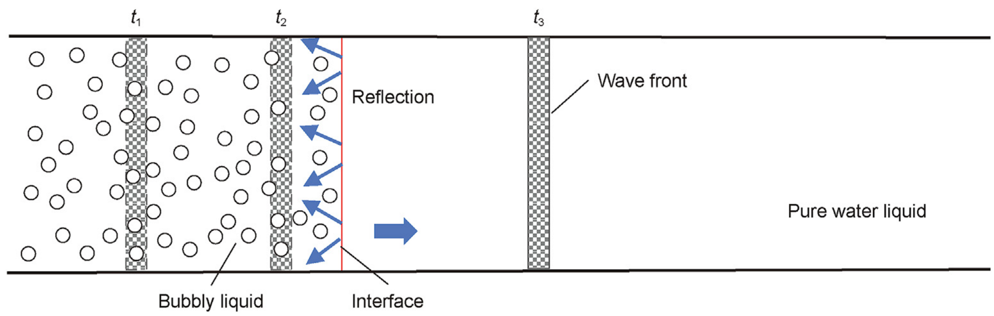


Fig. 17. The propagation of sound waves in bubbly liquid pipelines. The bubbles are non-uniformly distributed at the axial direction of the pipeline.

Fig. 7(a)–(d)). The theoretical calculation of the overall sound speed uses the physical concept of average speed, the length of the calculation field is L, the proportion of bubble axial distribution space is x, the proportion of pure water liquid space is $1-x$, the overall sound speed V is calculated as:

$$V = \frac{L}{t} \tag{18}$$

$$t = t_l + t_g \tag{19}$$

$$t_g = \frac{xL}{c} \tag{20}$$

$$t_l = \frac{(1-x)L}{a_l} \tag{21}$$

where $t = t_l + t_g$ is the total time for the sound wave in the calculation domain, t_l is the time in pure water liquid and t_g is the time in bubbly liquid.

Substituting Eqs. (19)–(21) into Eq. (18) obtains the formula for the overall sound speed when bubbles are partially distributed in the pipeline axis as:

$$\frac{1}{V} = \frac{x}{c} + \frac{1-x}{a_l} \tag{22}$$

Fig. 18 shows the overall sound speed when bubbles are partially distributed in the pipeline axial direction and the calculated results of Eq. (22) under the same conditions. As seen in Fig. 18, the simulation results are in high agreement with the calculated results of Eq. (22), with a maximum error of less than 5.2%. As Eq. (22) is established from the mathematical level, the effect of the medium interface is not considered, which fully illustrates that although the presence of the interface affects the sound pressure distribution in the pipeline (Xu, 2003), it has a negligible effect on the sound speed.

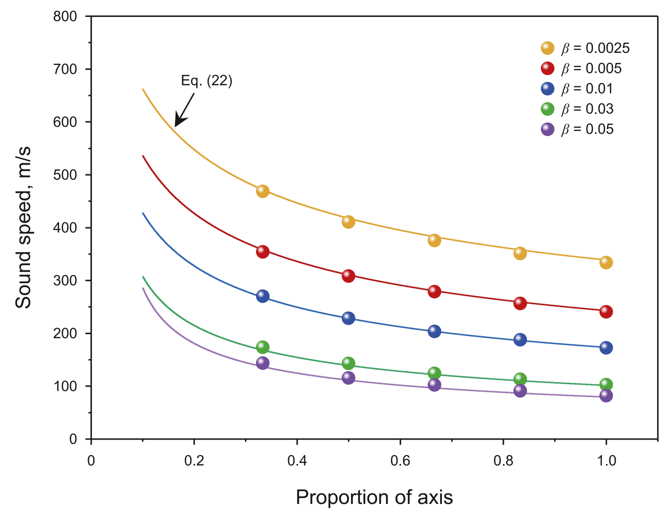


Fig. 18. Comparison of simulated sound speed with Eq. (22).

4.4.2. Non-uniform distribution of bubbles at the cross section of the pipeline

To investigate the effect of bubble distribution patterns in the pipeline cross-section on the overall sound speed, simulations of acoustic wave propagation were carried out with bubbles in 25%, 50%, 75% and 100% of the pipeline cross-section (see Fig. 6), the results are shown in Fig. 19. The results show that at a certain void fraction, the fluctuations in sound velocity for varying cross-sectional bubble distribution patterns are less than 0.4% of the average value, so the effect of the non-uniform bubble distribution in the pipeline cross-section on sound speed can be negligible.

Fig. 20 shows the propagation of an acoustic wave through a liquid pipeline containing bubbles that are not uniformly distributed in cross-section, which differs from Section 4.4.1 in that its propagation medium is equivalent along the propagation direction. According to the wave propagation mechanism, acoustic waves propagate as plane waves, and the studied wavelength of acoustic waves ($\lambda_{min} = 0.16$ m) is longer than the diameter of the pipeline ($D = 0.04$ m). Therefore, the void fraction of the medium in any micro-element volume is equivalent to the wave; even if the bubble distribution of the pipeline cross-section is not uniform, it can still

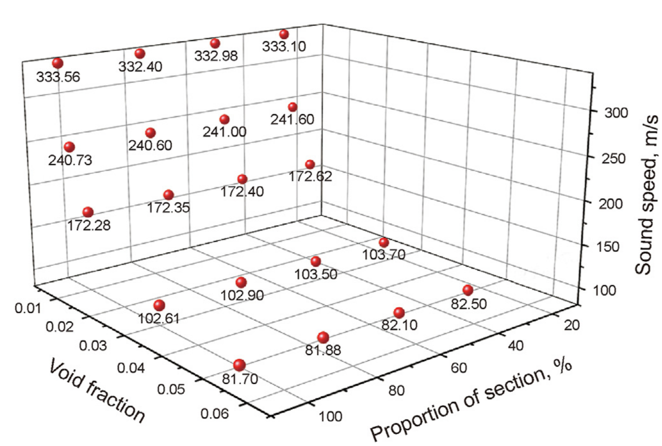


Fig. 19. Overall sound speed versus proportion of bubble cross-section distribution space.

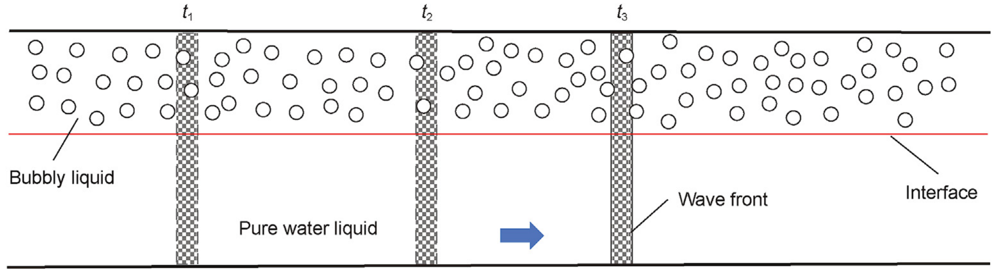


Fig. 20. The propagation of sound waves in bubbly liquid pipelines. The bubbles are non-uniformly distributed at the cross section of the pipeline.

be regarded as a homogeneous medium.

5. Conclusions

This paper is based on the technical challenges associated with the acoustic leak detection method in liquid pipelines containing gas bubbles. According to the signal characteristics of leakage sound waves, as well as the distribution patterns of bubbles in horizontal liquid pipelines, this study discusses the effect of acoustic frequency, bubble size, and bubble distribution patterns on the propagation characteristics of sound velocity. Moreover, the idealized theoretical sound speed model is selected and refined for horizontal pipe flows. The main conclusions of this paper are as follows.

(1) The sound velocity model based on the bubble dynamics can be used to calculate the propagation velocity of low-frequency sound waves and ultrasonic waves, but the model involves large number of complex equations, so the calculation process is more complicated. Based on the two-fluid model to establish the speed of sound model can be used to calculate the propagation velocity of low-frequency sound waves, and the calculation equation is simpler. Worth mentioning is that the results of two models are similar in the low-frequency range and for small bubble sizes.

(2) The energy of the acoustic signal from liquid pipeline leaks containing air bubbles is concentrated in the low frequency range (<200 Hz). In this case, the leakage hole is circular with a diameter of 6 mm and it is located directly beneath the pipeline. The research found that in the low-frequency range, acoustic waves propagate in bubbly liquids with almost no dispersion, so the two-fluid model is still highly accurate in investigating the propagation characteristics of low-frequency acoustic waves. Therefore, for the engineering applications, the most appropriate model for the sound speed is one which accounts for the two fluids.

(3) The speed of sound in bubbly liquid pipelines is not constant at a certain void fraction, which is related to the bubble size and distribution pattern. The sound speed remains almost constant and gradually increases as the bubble radius increases at low frequencies. The bubble size affects the gas-liquid heat transfer equilibrium, during which sound speed is affected. For this reason, a thermodynamic correction factor f_T is proposed to improve the calculation accuracy of the sound velocity for the two-fluid model with different bubble sizes.

(4) Sound waves propagate as plane waves in liquid pipelines containing gas bubbles. As a result, the non-uniform distribution of bubbles in the pipeline cross-section has a negligible effect on the overall sound speed of the pipeline. Instead, the overall sound speed increases non-linearly with the reduction of the bubble distribution space in the pipeline axial direction. This paper establishes a theoretical model to calculate the sound speed when the bubbles are non-uniformly distributed in the pipeline axial direction, which is in good agreement with the numerical calculation results. This is useful for the application of acoustic leak detection methods in liquid pipelines containing gas bubbles.

This research focuses on the propagation characteristics of small-amplitude simple harmonic waves in bubble-containing horizontal pipelines. In the future, the results of this paper need to be confirmed by more experimental studies, and the nonlinear propagation characteristics of finite-amplitude acoustic waves and the influencing factors need to be further investigated.

CRediT authorship contribution statement

Cui-Wei Liu: Writing – review & editing, Funding acquisition. **Lin-Jing Yue:** Writing – original draft, Methodology, Data curation. **Yuan Xue:** Validation, Software. **Shu-Fang Zhu:** Visualization. **Yu-Xing Li:** Writing – review & editing, Supervision.

Conflict of interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Appendix A. Supplementary data

Supplementary data to this article can be found online at https://doi.org/10.1016/j.petsci.2025.02.013.

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