Petroleum Science 22 (2025) 508–526

Contents lists available at ScienceDirect

Petroleum Science

journal homepage: www.keaipublishing.com/en/journals/petroleum-science

Review Paper

A survey on multi-objective, model-based, oil and gas field development optimization: Current status and future directions

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A R T I C L E I N F O

Article history:
Received 15 September 2023
Received in revised form
11 July 2024
Accepted 23 September 2024
Available online 25 September 2024

Edited by Jia-Jia Fei and Teng Zhu

Keywords:
Derivative-free algorithms
Ensemble-based optimization
Gradient-based methods
Life-cycle optimization
Reservoir field development and management

A B S T R A C T

In the area of reservoir engineering, the optimization of oil and gas production is a complex task involving a myriad of interconnected decision variables shaping the production system's infrastructure. Traditionally, this optimization process was centered on a single objective, such as net present value, return on investment, cumulative oil production, or cumulative water production. However, the inherent complexity of reservoir exploration necessitates a departure from this single-objective approach. Multiple conflicting production and economic indicators must now be considered to enable more precise and robust decision-making. In response to this challenge, researchers have embarked on a journey to explore field development optimization of multiple conflicting criteria, employing the formidable tools of multi-objective optimization algorithms. These algorithms delve into the intricate terrain of production strategy design, seeking to strike a delicate balance between the often-contrasting objectives. Over the years, a plethora of these algorithms have emerged, ranging from a priori methods to a posteriori approach, each offering unique insights and capabilities. This survey endeavors to encapsulate, categorize, and scrutinize these invaluable contributions to field development optimization, which grapple with the complexities of multiple conflicting objective functions. Beyond the overview of existing methodologies, we delve into the persisting challenges faced by researchers and practitioners alike. Notably, the application of multi-objective optimization techniques to production optimization is hindered by the resource-intensive nature of reservoir simulation, especially when confronted with inherent uncertainties. As a result of this survey, emerging opportunities have been identified that will serve as catalysts for pivotal research endeavors in the future. As intelligent and more efficient algorithms continue to evolve, the potential for addressing hitherto insurmountable field development optimization obstacles becomes increasingly viable. This discussion on future prospects aims to inspire critical research, guiding the way toward innovative solutions in the ever-evolving landscape of oil and gas production optimization.

1. Introduction

In modern society, despite technological advancements, the heavy reliance on scarce and finite fossil fuels like oil and gas persists as the predominant energy source. As the world's demand for petroleum keeps growing, global petroleum reserves are depleting due to continuous exploitation. Consequently, the oil-gas production industry is grappling with multiple challenges, such as increasing water content, low efficiency, and rising costs, which lead to the waste of resources and environmental pollution (Cordes et al., 2016). The oil industry's sustainable development has been negatively impacted by challenges such as reduced investment incentives due to high oil prices, emphasizing the importance of adopting advanced methods to improve margins considering unstable prices (Silva et al., 2015). Hence, optimization technologies have become vital in oil and gas production to improve productivity and reduce energy consumption (Morales et al., 2011; Nasir et al., 2022).

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https://doi.org/10.1016/j.petsci.2024.09.018

Field development faces the challenge of conflicting objectives that need to be optimized for consistent, accurate, and low-risk exploration. Traditionally, single-objective optimization algorithms have only considered economic gain (Rosenwald and Green, 1974). However, research shows that complex problems, like production optimization, require model-based optimization with multiple objectives. For instance, some research has shown that including environmental-related objective functions (e.g., CO_2 emission, voidage-replacement ratio, or cumulative water production) in the optimization procedure is crucial for a more sustainable and environmental-friendly exploration (Khan et al., 2013; Awotunde and Sibaweihi, 2014; You et al., 2019; Sun et al., 2021; Tom et al., 2024). In the face of this challenge, researchers have been investigating the application of multi-objective algorithms for model-based life-cycle production optimization under multiple, conflicting objectives. In the petroleum engineering literature, many multi-objective optimization implementations and applications have been published (Schulze-Riegert et al., 2007; Ferraro and Verga, 2009; Hajizadeh et al., 2011; Mohamed et al., 2011; Lu et al., 2013, 2017; Liu and Reynolds, 2014, 2015, 2016a, 2016b; Park et al., 2015; Hutahaean et al., 2017a, 2017b). Recently, there has been significant attention towards employing multi-objective algorithms for production optimization in field development problems.

Multi-objective optimization problems are complex due to the absence of a single, global optimum strategy. Instead, multi-objective optimization problems require the identification of a set of optimal strategies that represent the trade-offs between conflicting objective functions, known as Pareto front (Coello et al., 2007). The search space exploration for multi-objective optimization problems often requires a higher number of reservoir simulations, which increases computational cost and time for optimization. Although multi-objective optimization problems can be impractical for direct use in production optimization, recent advances in multi-objective optimization methods have improved optimization while reducing computational costs. These novel techniques have not been implemented in the context of field development and could be further explored to promote faster and more reliable optimization. Therefore, since the development of multi-objective algorithms that is designed specifically for production optimization and their complex nature is a major concern for the petroleum industry and scientists in the field, this survey aims to enumerate, classify, and analyze the existing contributions of multi-objective algorithms for production optimization. We also identify and outline open opportunities that may be vital in the development of suitable methods for practical, real-world applications in the future.

The remaining of this paper is organized as follows. Section 2 introduces production optimization and reservoir management. Section 3 presents the basic concepts of multi-objective production optimization. Section 4 reviews existing methods. Section 5 discusses open challenges. Section 6 discusses the future direction. Finally, Section 7 concludes this work.

2. Production optimization field development and reservoir management problem

This section introduces field development and management, life-cycle optimization, uncertainty handling, and the categorization of decision variables and objective functions for multi-objective production optimization.

2.1. General introduction to closed-loop field development and management

In field development and management, optimizing the production strategy is critical to maximize the profitability and productivity of the field (Rostamian et al., 2019b). The closed-loop field development (CLFD) approach is a coordinated, multidisciplinary approach to utilizing newly acquired information iteratively during field development to optimize production strategy. CLFD involves three steps: optimizing the field-development plan based on diverse sources of knowledge, drilling new wells and acquiring diverse types of data, and updating multiple representative geological models supported by accessible information. By implementing an optimization tool, new wells are optimized in terms of number, type, location, and controls (Shirangi and Durlofsky, 2015; Shirangi, 2019).

To develop a general methodology that incorporates field development and reservoir management, the closed-loop reservoir management (CLRM) approach has been proposed. CLRM is a closed-loop feedback process that uses updated data to adjust reservoir management decisions continuously (Jansen et al., 2009). Based on CLRM, the closed-loop field development and management (CLFDM) approach is proposed, which is an innovative field management and development method that involves updating existing field models periodically and optimizing production to maximize the value of the field economically (Schiozer et al., 2019). Fig. 1 illustrates the CLFDM. The significant stages of the workflow are categorized by color. Green represents data collection, evaluation of uncertainty, and model development. Blue shows the implementation of models to make long-term decisions under uncertainty. Red indicates data assimilation, which directly affects simulation models or high-fidelity geologic models to select models used in the blue section. Black demonstrates short-term production optimization and production strategy selection (Schiozer et al., 2019).

It must be mentioned that optimization of the production strategy is the main challenge in the blue part of the methodology developed by Schiozer et al. (2019). Further details can be found in (Schiozer et al. 2015, 2019).

2.2. Uncertainty handling

Production strategy optimization can be done using a single model with nominal optimization, which maximizes or minimizes an objective function. However, this approach does not consider uncertainty and may lead to poor performance in different situations. Alternatively, robust optimization involves optimization over an ensemble of models to find strategies that are less sensitive to model variability. This approach evaluates each strategy in multiple models but comes with a higher computational cost and decision analysis time (Yeten et al., 2004; van Essen et al., 2009b; Fonseca et al., 2014a, 2014b; Siraj et al., 2015; Yasari and Pishvaie, 2015; Fu and Wen, 2017a, 2017b; Lu and Reynolds, 2019, 2020; Mirzaei-Paiaman et al., 2021; Nguyen et al., 2023a; Davari et al., 2024).

2.3. Duration of field development optimization

The duration of field development optimization can be long or short-term. Long-term or life-cycle optimization aims to maximize the net present value (NPV) of production over the entire lifespan of the field, which can be enhanced by integrating robust optimization and risk minimization. Short-term optimization, on the other hand, focuses on maximizing revenue in the near term and is often neglected in studies on life-cycle performance. However, due to uncertainties, short-term objectives often determine the operational strategy and integrating them into life-cycle optimization is crucial for closed-loop reservoir management (Chen et al., 2012; Yasari et al., 2013; Zhao et al., 2019; Santos et al., 2021).

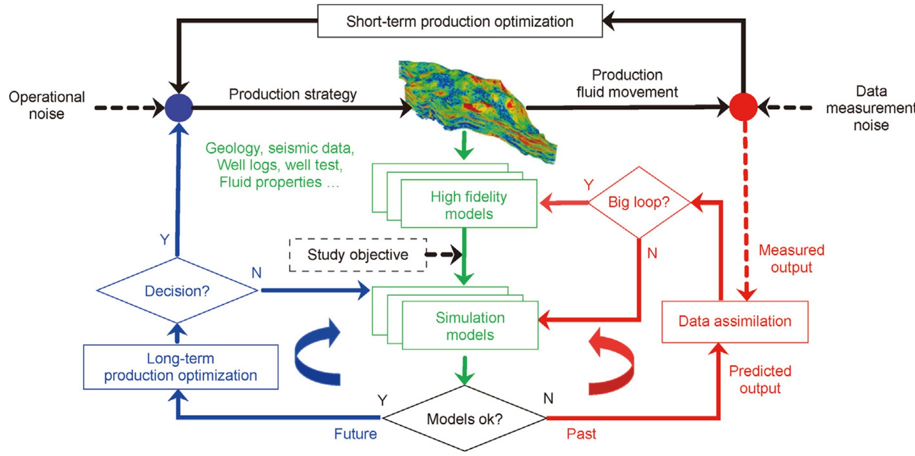

Fig. 1. Closed-loop field development and management (CLFDM). Source: (Schiozer et al., 2019).

2.4. Decision variables

Decision variables are used in production optimization to maximize or minimize objective functions. Decision-making concerning decision variables relies on aspects such as project nature, geographical setting, recovery mechanisms, and well completion types. The optimization variables may be classified into three groups: Group 1 (design variables), Group 2 (operational variables), and Group 3 (future variables). Design variables require significant investment and include recovery method, well type, number of wells, well pattern, and well location. Operational variables measure the system and include control valve choke at various levels. Future variables determine potential future scenarios and include activities such as infill drilling, recompletion, and well conversions (Barreto and Schiozer, 2015; Barreto et al., 2016; Gaspar et al., 2016).

2.5. Objective functions

In the production strategy optimization problem, objective functions evaluate the efficiency of a solution. Objective functions are computable functions of decision variables, which may be commensurable or non-commensurable (Coello et al., 2007). There are several objective functions in petroleum production strategy optimization, including economic-based functions such as NPV, risk, and return over investment, as well as field response-related functions such as water production, oil production, gas production, water breakthrough time, and recovery factor. Objective functions related to the field's injection process, such as water injection, gas injection, and polymer injection, can also be considered. The statistical manipulation of each objective function can be assumed as a predefined objective function in the context of uncertainty reduction, such as standard deviation, variance, and average. Short-term objectives and long-term strategies are often conflicting, making it important to balance nominal and robust objective functions (van Essen et al., 2009a; Chen et al., 2012; Murphy, 2014; Pearce, 2016; Schiozer et al., 2017; Santos et al., 2017, 2021; Rostamian et al., 2024). Fig. 2 shows how the objective functions are categorized based on their types in accordance with what has already been explained.

3. Multi-objective production optimization

Multi-objective production optimization problems are the ones containing multiple objectives that must considered simultaneously. The general definition of multi-objective optimization is as follows:

Assume that $f_1(x), f_2(x), \ldots., f_k(x)$ is the k objective functions that need to be minimized or maximized and $x = (x_1, x_2, \ldots, x_n)$ represent the vector of decision variables. There are two types of constraints: equality constraints $g_i(x) = 0$ and inequality constraints $h_j(x) \le 0$. The formulation can be expressed as below in Eq. (1):

$$\text{Maximize/Minimize}: F(x) = (f_1(x), f_2(x), \ldots., f_k(x))$$

$$\text{Subject to}: \; g_i(x) = 0, i = 1, 2, \ldots, m$$

$$h_j(x) \le 0, j = 1, 2, \ldots, p \tag{1}$$

$$x \in \Omega$$

where Ω is a feasible region defined by the constraints.

Multi-objective production optimization problem is subject to conflicting objective functions, that is, it becomes impossible to improve one objective function without decreasing the other. Therefore, in multi-objective optimization problems, there is not only a single optimal strategy, but several optimal strategies that represents the trade-offs between the objectives. The set of optimal strategies is called the Pareto set. The concept of Pareto optimality is described below, according to (Coello et al., 2007).

Definition 1. (Pareto optimality) A production strategy $x\rightarrow$ is a Pareto optimal solution of the feasible region Ω if and only if (iff) there is no other production strategy $y\rightarrow \in \Omega$ where $F(y\rightarrow)$ dominates $F(x\rightarrow)$.

As can be seen, to identify which strategy is better than another in a multi-objective production optimization problem, they must be evaluated using the concept of Pareto dominance, detailed below.

Definition 2. (Pareto dominance) Given two production strategies $x\rightarrow \in \Omega$ and $y\rightarrow \in \Omega$, $x\rightarrow$ dominates $y\rightarrow$ if $x\rightarrow$ is no worse than $y\rightarrow$ in all objectives and $x\rightarrow$ is strictly better than $y\rightarrow$ in at least one objective, that is, supposing a minimization problem (Eq. (2)):

$$\vec{x} \prec \vec{y} \Leftrightarrow \forall m \in \{1, \ldots, M\}, f_m(\vec{x}) \le f_m(\vec{y})$$

$$\exists i \in \{1, \ldots, M\}:$$

$$f_i(x\rightarrow) < f_i(y\rightarrow) \tag{2}$$

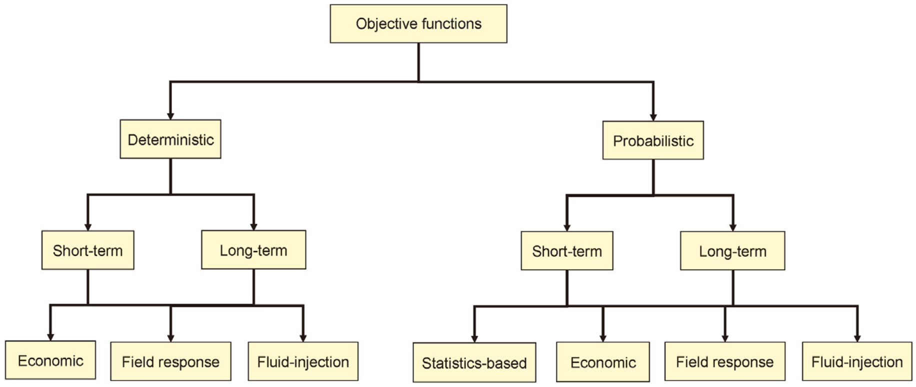

Fig. 2. Objective functions classification.

The Pareto set is composed of the optimal strategies within the entire feasible region. These strategies are not dominated by any other feasible strategy, as defined below.

Definition 3. (Pareto set) A Pareto optimal set is defined as (Eq. (3)):

$$\text{Pareto set} = \{\vec{x} \in \Omega \,|\, \neg\exists\ \vec{y} \in \Omega\ F(\vec{y}) < F(x \rightarrow)\} \tag{3}$$

To facilitate further analysis, it is common to generate the Pareto front. The Pareto front represents the set of points from the Pareto set displayed in the objective space. The formal definition of the Pareto front is presented below.

Definition 4. (Pareto front) Given a Pareto optimal set, the Pareto front is defined as (Eq. (4)):

$$\text{Pareto front} = \{F(x\rightarrow)\,|x\rightarrow\ \in PS\} \tag{4}$$

Here, it is important to distinguish that a multi-objective production optimization problem operates in two different spaces: the decision and the objective space. Fig. 3 explains both spaces.

4. Multi-objective production optimization in field development optimization

Despite their widespread use in other engineering fields, petroleum engineering problems remain largely untapped by multi-objective optimization approaches. Fig. 4 shows the number of publications per year about multi-objective production optimization problems. As can be seen, the interest of the research community has grown considerably in the past decade. Please note that these papers were obtained through different scientific databases to which the authors had accessed, including Web of Science, IEEE Xplore, Science Direct, SpringerLink, Scopus, and OnePetro. In addition, the search range is from 1980 to 2024. Papers outside of this scope could not be obtained and, therefore, were not considered for this review.

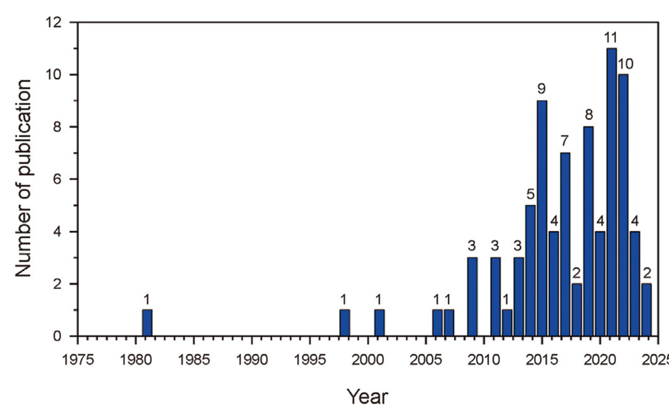

Fig. 4. Publications per year about multi-objective production optimization problem.

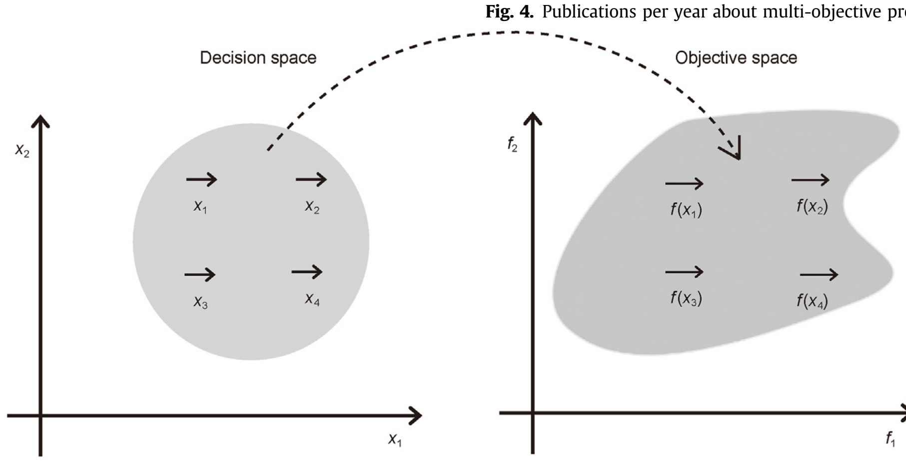

Fig. 3. A hypothetical bi-objective problem to illustrate the two different spaces in a multi-objective production optimization. Here, each production strategy $x\rightarrow_i$ is composed of two decision variables x_1 and x_2. All possible values for x_1 and x_2 compose the feasible decision space (left). Each production strategy $x\rightarrow$ is evaluated, and their specific set of objective function values is plotted into the feasible objective space (right). Feasible regions are shaded in grey. Source: created by the authors.

The research literature for multi-objective production optimization problem has been done considering two main approaches, as shown in Fig. 5: a priori, where single-objective methods are employed in a hierarchical approach or using a linear combination of functions (simultaneously), and a posteriori, where Pareto-based algorithms are employed. Fig. 6 shows the percentage of papers for each category. In this section, multi-objective optimization studies in the field development and management are reviewed considering the mentioned approaches.

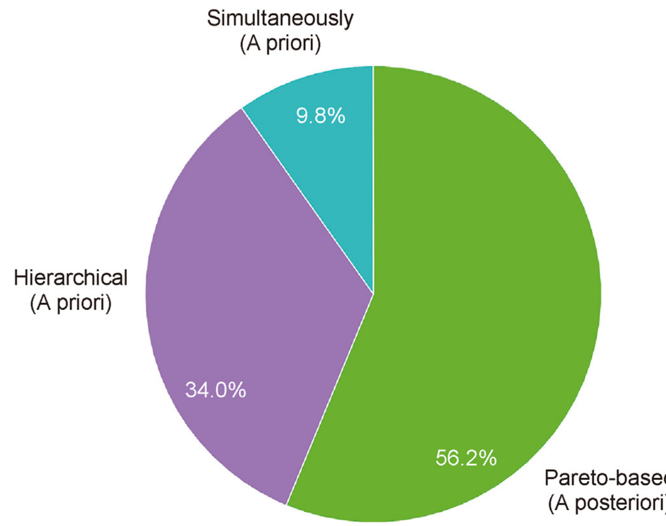

Fig. 6. Percentage of papers of each category.

4.1. A priori methods

A priori methods require that predefined components be defined before the optimization takes place. The two main approaches for this are priori-simultaneously, which forms a linear combination of functions (weighted sum), and priori-hierarchical, where the primary and secondary functions are predefined, and the secondary functions are optimized considering constraints with a certain degree of allowable changes in the primary functions. These approaches are detailed in the next subsections.

4.1.1. Priori-simultaneously

The necessity for handling many objectives in petroleum engineering has been emphasized since 1990s. The study of Harrison and Tweedie (1981) was among the first research in the area of field development with multiple objectives. Using an analytical approach, they incorporate linear combination of conflicting criteria with predetermined importance to select the most effective production policy.

Since then, many other works considered the same linearization approach, for instance, to maximize oil recovery and minimize investments (Xiao et al., 1998), maximize both production and NPV (Rahman et al., 2001), maximize minimum or mean NPV while minimizing NPV variance over an ensemble of geological realizations (Bailey et al., 2005; Capolei et al., 2015; Liu and Reynolds, 2014, 2015, 2016a, 2016b; Lu et al., 2017), minimizing water injection while maximizing oil production in water flooding plans (Cardoso, 2009), maximizing short-term oil rates and long-term recovery (Khan et al., 2013), maximize both short and long-term NPV (Hasan et al., 2013), maximizing NPV and minimizing risk (Isebor et al., 2014a, 2014b), maximizing NPV while minimizing voidage replacement ratio (Awotunde and Sibaweihi, 2014), minimize water cut while also minimizing gas production (Hanea et al., 2019), maximizing return over the investment and minimizing risk (Christiansen et al., 2016), maximizing both short and long-term NPV (Christiansen et al., 2017), undiscounted and highly discounted NPV (Fonseca et al., 2016), or by applying other combinations or statistic-based functions based on these mentioned objective functions (Siraj et al., 2016; Pinto et al., 2019; Wang et al., 2021c, 2022b; Alpak et al., 2022).

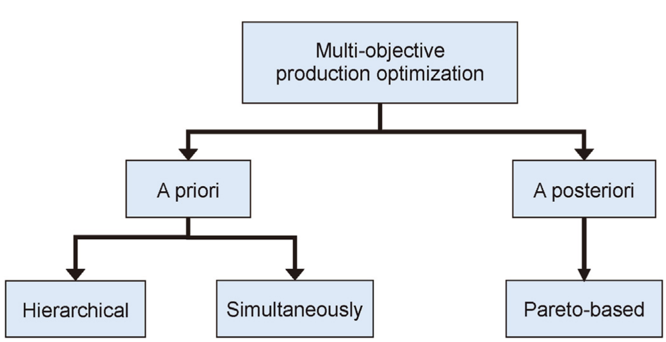

Fig. 5. Approaches to handle multi-objective production optimization problem.

Priori-simultaneously approach has the advantage of combining multiple functions into a single linear combination, which allows the application of single-objective optimization algorithms. The main concern when using this approach is that aggregation functions approaches have difficulties in finding a well-distributed set of solutions over non-convex Pareto front (Das and Dennis, 1997). In addition, they also require extensive studies to identify the optimal weights for a given problem, which can be considered an additional source of uncertainty.

4.1.2. Priori-hierarchical

In hierarchical optimization, the optimization of secondary objectives is bounded by a threshold of allowed change in the primary objectives (Fonseca et al., 2014b). The study by van Essen et al. (2009a) indicates that optimizing production over the long term can be a challenging problem with no clear and unique solution; more degrees of freedom in the control variables could still be an issue. Using additional degrees of freedom in the estimation of control variables, van Essen et al. (2009a) proposed a hierarchical approach to this issue. According to van Essen et al. (2011), their previous method was theoretically robust but computationally problematic for real-world-sized problems because it depends on determining the objective's Hessian matrix. Therefore, they devised an alternative approach that prioritizes the optimization of primary and secondary objective functions in an alternating order. As part of the second method, gradient information is used to prove the existence of a redundant degree of freedom regarding a life-cycle objective function. They argued that rather than using gradient-based optimization methods to tackle hierarchical optimization problems, alternative optimization methods (e.g., genetic algorithms) are possible.

Also, Chen et al. (2011, 2012) reported that the methodology of van Essen et al. (2009a) involved the determination of the Hessian matrix is a computationally demanding process. Consequently, they introduced a two-stage production optimization workflow. The first stage involved addressing the life-cycle constrained optimization problem, while the second stage focused on optimizing the short-term NPV under the imposed constraint. To incorporate the long-term NPV constraint along with the physical constraints into the short-term optimization, they employed the augmented Lagrangian method.

Fonseca et al. (2014a, 2014b) developed a robust gradient formulation as a modification of the hierarchical optimization workflow which improved the computational efficiency and examined the effectiveness of ensemble optimization (EnOpt) for robust multi-objective optimization on a hypothetical test case. They found that EnOpt is a computationally superior approach when the adjoint is inaccessible, in particular for robust optimization. Later, Fonseca et al. (2015) consider the same approach for the on-and-off inflow-control devices problem. They employ a switching-time reparameterization to address the limitation of gradient-based techniques in effectively handling discrete control problems.

The study conducted by Siraj et al. (2016, 2017) explored an optimal life cycle optimization solution that aimed to strike a balance between short and long-term gains. Additionally, the solution aimed to ensure the robustness of economic and geological uncertainties, particularly in relation to the predicted NPV. The authors use the work of van Essen et al. (2011) and Fonseca et al. (2014a, 2014b) as a base example. They claimed that compared to multi-objective optimization, the weighted sum approach decreases the impacts of uncertainty when implemented with mean-variance optimization objectives. Zhao et al. (2019) developed the augmented Lagrangian function to address the optimization of short and long-term NPVs while considering both linear and nonlinear constraints. This study uses two unique versions of the general stochastic approximation (GSA) algorithm: simultaneous perturbation stochastic approximation (SPSA) and EnOpt.

Nguyen et al. (2022) optimize CO_2 storage and recovery in a depleted oil reservoir using a stochastic gradient-based framework. The study employs line-search sequential quadratic programming (LS-SQP) with stochastic simplex approximated gradients (StoSAG) to maximize both net present value (NPV) and net present carbon tax credits (NPCTC), demonstrating computational efficiency and effectiveness in handling nonlinear constraints and estimating the Pareto front. Nguyen et al. (2023b) explore carbon capture, utilization, and storage (CCUS) in a CO_2-enhanced oil recovery process, focusing on maximizing NPV and NPCTC while respecting injection bottom hole pressure (IBHP) constraints. The study uses a lexicographic method based on LS-SQP with StoSAG, showing computational efficiency and highlighting the benefits of bi-objective over single-objective optimization. Nguyen et al. (2023c) continue this research on optimizing CCUS in CO_2-enhanced oil recovery, using LS-SQP with StoSAG to maximize NPV and NPCTC. The study effectively handles constraints, achieves a 13.8% increase in NPCTC with minimal NPV trade-off, and constructs a Pareto front to illustrate the trade-offs between objectives. Several studies have explored the application of multi-objective StoSAG optimization within the context of field development, highlighting its effectiveness in balancing competing objectives. By employing StoSAG optimization, these studies demonstrate how advanced computational techniques can significantly enhance strategic planning and operational decisions in complex field development scenarios (Fonseca et al., 2015, 2016; Lu et al., 2017, 2023; Liu and Reynolds, 2021; Nguyen et al., 2022, 2023a, 2023b).

Although common in the petroleum industry, the hierarchical approach has its own drawbacks. Single-objective methods lack the capability to identify multiple strategies in a single run, and even after multiple executions, there is no assurance of obtaining an evenly distributed set of strategies (Schulze-Riegert et al., 2007). Besides, single-objective methods may perform poorly in complex Pareto front geometries such as in non-convex, discontinuous, and non-smooth fronts. Finally, they often require a manual labor of constrained definition throughout the independent executions.

4.2. *A posteriori methods*

The a posteriori method allows decision-makers (DMs) to examine the performance of a problem and select one of the solutions that best meet their objectives. Over the past decade, meta-heuristics became a hot research topic for solving a posteriori multi-objective production optimization as they are able to produce high-quality solutions. Meta-heuristics simplicity has made them widely studied in petroleum engineering research problems. The purpose of this section is to elaborate on the application of this type of multi-objective approach to production optimization problems.

Ray and Sarker (2006), as one of the earliest Pareto-based multi-objective optimization studies, applied the Non-dominated sorting genetic algorithm II (NSGA-II) to the gas-lift problem with the goal of minimizing injected gas and maximizing produced oil in gas lift operation. Later, based on the multi-objective genetic algorithm (MOGA), Min et al. (2011) proposed a numerical scheme to optimize the number of injectors and their location in a water-flooding project where some producers are already present. Maximizing revenue and minimizing capital expenditures (CAPEX) and operating expenditures (OPEX) are the two main objectives implemented in their study.

Yasari et al. (2013) optimized well controls involving objective functions that maximize or minimize components of NPV. The optimization was performed with NSGA-II. Later, Yasari and Pishvaie (2015) presented a formulation to find robust water injection policies in a field development problem. In a study by Chang et al. (2015a, 2015b), the mean-variance approach was applied to estimate well placement based on geological uncertainties using NSGA-II. They demonstrated that oil in place is an essential indicator for well placement and that quality map-assisted NSGA-II (QM-NSGA-II) can be significantly beneficial for optimizing well placement.

Plaksina and Gildin (2015) developed a method for shale gas reservoir development optimization to reduce development risks while maximizing economic gains. Their research was conducted on hydraulic fracturing optimization of shale gas wells using NSGA-II. Safarzadeh et al. (2015) used a genetic algorithm in conjunction with the streamlined simulation approach to enhance the injection efficiency. Compared to traditional techniques such as weighted sums, the proposed methodology resulted in increased oil production, reduced water cuts, and lower water loss to aquifers.

A study by Bagherinezhad et al. (2017) explored the potential for optimizing the oil well placement and control of wells in naturally fractured reservoirs. They proposed an optimization framework based on NSGA-II and applied it for well placement and control problems to maximize cumulative oil production and water breakthrough time. Using uncertain multi-objective programming models, Ji et al. (2017) formulate an optimization problem for the oilfield development plan by considering uncertain geological, technical, and economic factors. Concerning oil production and resource limitations, it minimizes expected development costs while maximizing anticipated new recoverable reserves. Rostamian (2017) implemented a modified NSGA-II to optimize the NPV and recovery factor in a synthesized model considering the type of well, the number of wells, and the location of the well as the decision variables. He demonstrated that decisions-making variables have a significant impact on the nature of optimization problems.

Rostamian et al. (2019a, 2019b) utilized similarity-based and non-dominated ranked genetic algorithms (NRGAs) to address the water flooding multi-objective optimization problem. Their articles claimed to have fully generated a Pareto front, which offers several production strategies for optimizing field development. Moradi and Rasaei (2017) presented a model of automated reservoir management, which incorporated the situation awareness technique as a method that was able to define multi-objective optimization problems regarding reservoir dynamic situations pertaining to economic, technical, and operational managerial targets. The multi-objective algorithm employed in this study was NSGA-II.

Fu and Wen (2017a, 2017b) developed a framework to appropriately select a solution for field development problems that could efficiently deal with different multi-objective optimization issues. As an example, the adjoint-based method, MOGA, multi-objective particle swarm optimization (MOPSO), and their hybridization at multiple scales are included. Later, Fu and Wen (2018) compared multi-objective optimization solutions to a conventional single-objective solution. According to their study, multi-objective optimization is capable of offering a plan to balance oil and water production. However, the weighted sum strategy used in single objective optimization is less rigorous and combines multiple metrics. Camara et al. (2018) performed an examination of three different multi-objective optimization algorithms, namely NSGA-II, the greedy randomized adaptive search procedure, and the ε-Constraint method. According to their study, the objective function was the maximization of oil production while minimizing costs and environmental damage. As a result of their research, they found that the NSGA-II method had a lower computational cost than other methods.

Hutahaean et al. (2019) developed a well placement optimization framework under uncertainty in a Bayesian framework. The workflow selects Pareto models based on a clustering algorithm from a set of history-matched models designed to capture geological uncertainty. The optimal solutions are selected based on a probabilistic risk analysis of the selected models using MOPSO. The surrogate-assisted reference vector evolutionary algorithm principal component analysis method (SA-RVEA-PCA) proposed by Zhao et al. (2020a, 2020b) is designed to solve robust (ensemble-based) production optimization problems. It is based on the decomposition-based algorithm called the reference vector evolutionary algorithm (Cheng et al., 2016). It employs the Gaussian process (also known as Kriging) surrogate model to replace the computationally expensive reservoir simulations and PCA to reduce the dimensionality of the original problem.

Wang et al. (2021a, 2021b, 2022a) developed three methods for well control optimization using NSGA-II and proxy models. The aim was to increase cumulative oil production and revenue. Support vector regression, tri-training surrogate-assisted, and self-adaptive multi-fidelity surrogate-assisted models were conducted in their study.

Liu et al. (2021) developed an error compensation model of the oil-gas production process for optimizing three indicators: oil production, water production, and energy consumption per ton of oil. Furthermore, to improve the performance of solutions, an improved NSGA-II with multi-strategy improvement (IMS-NSGA-II) was proposed to solve the error compensation of the original model (Liu et al., 2021). Based on a field case study, Zhang et al. (2021) developed a method that optimizes inflow control device configurations in horizontal wells by considering water and sand control factors. Based on the proposed method, NSGA-II was utilized as the optimizer, and water breakthrough time and production rate were considered optimization objectives. Based on their findings, the NSGA-II can play a significant role in water and sand control in horizontal wells.

The study by Yan et al. (2021) uses a robust optimization approach to solve long-term strategic decision-planning problems under uncertainty. To reduce costs and enhance sustainable development, the authors construct a multi-objective, multistage, robust integer optimization model with NSGA-II. A study by Al-Aghbari et al. (2021) used the NSGA-II algorithm to investigate distinct functions for the short and long-term management of flooding in the Brugge field. According to their studies, cumulative oil production, water cut, cumulative water production, and NPV are the main objectives. In a follow-up study, Al-Aghbari et al. (2022a, 2022b), used the same methodology. However, instead of using a reservoir simulator, they applied a proxy model, an evolutionary neural network, to model the reservoir.

Farahi et al. (2021a) applied MOPSO to address water flooding in a synthetic reservoir under geological uncertainty. For the conservative production plan, they maximized short and long-term NPV simultaneously. In their study, *k*-means clustering is implemented to reduce realizations, reducing computational costs. In another study, the authors used MOPSO and NSGA-II methods to simultaneously identify short and long-term strategies in water-flooded reservoirs (Farahi et al., 2021b). In their study, modifications have been made to the algorithms to address the imperfect Pareto front obtained for multi-objective optimization problems. They stated that based on a comparison of various methods, MOPSO provided a more appropriate production strategy for optimizing short term and long term NPV in both cases than NSGA-II did. Later, Farahi et al. (2022) applied the same methodology (MOPSO) for the reservoir with a gas condensate problem.

The work by de Moraes and Coelho (2022a) proposes a combination of multi-Objective evolutionary algorithm based on decomposition and novel-first tabu search (MOEA/D-NFTS), a diversity preservation-guided method that employs local search methods with knowledge incorporation to avoid duplicated strategies in production optimization problems with discrete decision variables. The authors compared the proposed design with a baseline algorithm (MOEA/D) in a well placement optimization problem using the UNISIM-I-D benchmark case, considering the maximization of cumulative oil production and minimization of cumulative water production as the objective functions. They showed that incorporating diversity preservation mechanisms produces statistically significant superior results in comparison to optimization without diversity preservation in production optimization with discrete decision variables.

4.2.1. Advantages and disadvantages of current a posteriori methods

A posteriori multi-objective production optimization has been performed mainly considering four algorithms: NSGA-II, MOPSO, RVEA, and MOEA/D-NFTS. Table 1 presents a summary of the advantages and disadvantages of these methods. NSGA-II has a very simple structure and easy implementation, can be used for both continuous and discrete variables by applying specific genetic operators, and is known to have a fast convergence (Deb et al., 2002); crowding distance guarantees diversity in the objective space, it achieves good results in many regular and irregular Pareto front geometries and can be easily adapted to different mating schemes, which may improve even further its performance. As for disadvantages, NSGA-II is not scalable to many objective problems; it is highly sensitive to the initial population (Poles et al., 2009); crowding distance does not guarantee diversity in the decision space (as only the objective functions values are used for density estimation); and it is unsuitable for user preference articulation in its original form, as there is no way to predetermine a region that the DMs want to be explored.

MOPSO has a fast convergence, a simple structure, and is less sensitive to the initial population than NSGA-II. As for drawbacks, it has a high risk of falling into local optima (Li et al., 2021; Ünal and Kayakutlu, 2020), is designed for continuous variables only, particles may continuously move outside the boundaries, which affects the search space exploration (Pang and Ng, 2018), it is not scalable for many objective problems, and is unsuitable for user preference articulation.

Table 1
Advantages and disadvantages of Pareto-based multi-objective algorithms found in the literature of multi-objective production optimization.

Algorithm	Advantages	Disadvantages
NSGA-II	- Simple structure - Guarantees diversity in the objective space - Easily adapted to different mating schemes - Easily adapted to different mutation schemes - Fast convergence - Handles continuous and discrete variables - Satisfactory results in complex Pareto front geometries	- Not scalable for many-objective optimization - Does not guarantee diversity in the decision space - Unsuitable for user preference articulation in its original form
MOPSO	- Fast convergence - Have few hyper-parameters to tune - Simple structure - Less sensible to the initial population - Guarantees diversity in the objective space	- Considerable risk of falling into local optima - Originally designed for continuous variables only - Particles may continuously move outside boundaries - Poor local search ability - Does not guarantee diversity in the decision space - Not scalable for many-objective optimization - Unsuitable for user preference articulation in its original form
RVEA	- Scalable to many-objective optimization - Stable performance in different problems - Reliable results in complex Pareto front geometries - Suitable for user preference articulation - Guarantees diversity in the objective space - Handles continuous and discrete variables - Easily adapted to different mating schemes - Easily adapted to different mutation schemes - Fast convergence	- Normalization causes instability in convergence - More complex structure - Does not guarantee diversity in the decision space
MOEA/D-NFTS	- Scalable to many-objective optimization - Stable performance in different problems - Reliable results in complex Pareto front geometries - Suitable for user preference articulation - Guarantees diversity in the objective space - Easily adapted to different mating schemes - Easily adapted to different mutation schemes - Incorporates domain knowledge into the optimization - Fast convergence - Produces a well-diversified set of strategies	- Requires adaptations according to each problem - More complex structure - Designed for discrete variables only - Requires some sort of prior knowledge

RVEA is a robust multi-objective algorithm due to its scalability to many objective problems. It has achieved efficient results and fast convergence for linear, non-linear, unimodal, multimodal, continuous, and discontinuous Pareto front geometries. RVEA also showed stable performance in different problems using the same hyper-parameter values, which shows it is less dependent on the user input parameters (Cheng et al., 2016). It also incorporates user preference articulation as the reference vectors can be distributed in the objective space according to DMs preferences. As possible drawbacks, RVEA has a more complex structure than other methods; objective normalization causes instability in the convergence (Cheng et al., 2016), so another adaptation method should be included (i.e., see the adaption method proposed in the original paper by Cheng et al. (2016), and it does not guarantee diversity in the decision space in combinatorial problems).

Finally, MOEA/D-NFTS also uses a decomposition-based structure, which makes it scalable to many-objective optimization. It has shown efficient results and fast convergence for linear, non-linear, unimodal, and multimodal problems, with different Pareto front geometries (de Moraes and Coelho, 2022b). It is suitable for user preference articulation, it can be easily adapted to different mating and mutation schemes, and since it incorporates a diversity preservation mechanism, it guarantees diversity not only in the objective space but also in the decision space, which produces an evenly distributed set of strategies at the end of the optimization. This can be achieved through the incorporation of domain knowledge into the optimization. As disadvantages, MOEA/D-NFTS is designed for problems with discrete decision variables, it requires adaptations according to each problem, it has a more complex structure, and some sort of prior knowledge is needed to build the so-called knowledge-assisted local search mechanisms.

5. Limitation and challenges

This section explores the complex limitations of existing multi-objective optimization frameworks in the context of optimizing petroleum production. Moreover, it discusses the mathematical challenges associated with the convergence problem in these algorithms and suggests potential solutions to overcome these limitations.

5.1. Limitations in multi-objective production optimization

A comprehensive understanding of multi-objective optimization algorithms utilized in petroleum production optimization requires acknowledging their inherent limitations. Although numerous studies have investigated their application in enhancing production efficiency and decision-making, a critical analysis of their constraints and shortcomings remains elusive. Addressing this gap is crucial for two primary reasons: firstly, it deepens our comprehension of the real-world challenges encountered in the petroleum industry, and secondly, it offers valuable insights for refining existing methodologies and crafting optimization strategies tailored to the industry's complexities. The subsequent discussion will address some of the most significant limitations.

5.1.1. Complexity of reservoir models

Petroleum reservoirs represent highly intricate systems characterized by a multitude of interacting factors. A reliable reservoir model may comprise millions of grid cells, necessitating hours or even days for simulation. Despite this complexity, many studies reviewed in this paper have utilized relatively simple models primarily for methodological testing purposes. Consequently, there exists a significant gap in the literature pertaining to the study of more complex reservoirs. Notably, only one study by Hanea et al. (2019) has examined a reservoir with over 300,000 grid cells, highlighting the limited exploration of such complexities. Commonly used models include the SPE10 model (Christie and Blunt, 2001), the Brugge test model (Peters et al., 2009), the Egg model (Jansen et al., 2014), and the PUNQ model (Floris et al., 2001), as depicted in Table 2, highlighting the models employed across various studies.

To address this gap, several approaches can be employed to mitigate the challenges associated with complex reservoir modeling. One promising avenue involves the application of physics-informed machine learning methods, which have emerged as a novel tool for reducing complexity while preserving the essential properties of the original model. By leveraging these advanced techniques, researchers can navigate the intricacies of complex reservoir systems more effectively, thereby enhancing our understanding of petroleum reservoir dynamics and facilitating the development of robust optimization strategies. Section 6.2 provides detailed information.

5.1.2. Scalability to large-scale problems

Optimizing production across numerous wells or fields presents a formidable challenge. Existing multi-objective optimization algorithms often struggle to "keep up" with the complexity and scale of such large-system applications. Addressing this scalability issue requires multi-pronged approaches. Decomposing the large-scale problem into smaller, manageable sub-problems could be explored. Leveraging parallel computing could offer substantial computational speedups (Al-Mouhamed et al., 2024; Mira et al., 2023; Rigon et al., 2024). Additionally, investigating hybrid algorithms that combine the strengths of different multi-objective optimization approaches tailored to specific problem aspects might be fruitful (Parashar et al., 2023).

Furthermore, incorporating domain knowledge into the algorithms through surrogate models or reduced-order models could significantly reduce computational burden while maintaining acceptable accuracy (Zhao et al., 2024). Lastly, developing specialized multi-objective optimization algorithms specifically designed for the unique challenges of petroleum production systems presents a long-term solution path. It should be noted that tackling real-world problems like these often requires a combination of approaches, and the optimal solution depends on the specific context and resources available. Evaluating and adapting these potential solutions in collaboration with domain experts is crucial for unlocking the full potential of multi-objective optimization in large-scale petroleum production optimization.

5.1.3. Limited practical validation

Despite existing research on multi-objective optimization for petroleum production optimization, a significant gap exists in the translation of these methods to real-world implementation and validation. This lack of practical validation raises concerns about their effectiveness and applicability in actual field settings. While studies often propose solutions promising increased profitability and productivity, a crucial gap remains in the absence of research following a robust methodology in real-world cases and reporting long-term production outcomes. This lack of post-implementation evaluation hinders the industry's adoption of multi-objective optimization methodologies and leaves their superiority and advantages unproven. Consequently, much of the research in this area remains theoretical, and their real-world application, if any, lacks documented evidence of their effectiveness.

5.1.4. Integration with existing workflows

The integration of optimization algorithms with existing engineering workflows and software tools presents a notable challenge within the industry. This difficulty is compounded by the fact that the non-free lunch theorem holds true for each unique case in reservoir engineering, as every reservoir possesses its own distinct set of challenges and complexities. Consequently, there is no universally applicable global optimization methodology that can seamlessly address the intricacies of multi-objective optimization problems across all scenarios. However, the authors suggest the adoption of a standardized language concerning optimization approaches, which could facilitate the translation of any optimization methodology to accommodate new fields or objective functions with varying optimization variables. This standardized framework should aim to streamline the integration process and enhance interoperability between different optimization techniques, thereby promoting greater efficiency and effectiveness in addressing multi-objective optimization challenges within the petroleum industry.

5.1.5. Handling uncertainty

Petroleum production systems inherently harbor uncertainty due to geological variability, reservoir property heterogeneity, and dynamic operational conditions. Regrettably, many multi-objective optimization algorithms utilized in this sector face challenges in adequately incorporating and managing such uncertainties. Surprisingly, 45% of the studies reviewed in this paper have exclusively employed single reservoir or economic models, representing a less sophisticated real-world scenario. However, lacking post-research support, these studies have not tested their models on an ensemble of reservoir or economic models. This deficiency often yields solutions ill-suited for real-world complexities, thereby potentially compromising their feasibility and economic viability.

Recently, various approaches have emerged to mitigate uncertainty in reservoir realizations. Data space inversion, learning-based data-driven forecast approaches, and RMfinder are among the tools that can be utilized for this purpose in multi-objective field development optimization. Although these methods have demonstrated applicability in simple to moderately sophisticated

Table 2
Reservoir models implemented multi-objective optimization field development studies.

Reservoir models	Authors
Egg model	van Essen et al. (2009a, 2009b, 2011); Fonseca et al. (2014a); Yasari and Pishvaie (2015); Fonseca et al. (2016); Siraj et al. (2017); Rostamian (2017); Rostamian et al. (2019a, 2019b); Pinto et al. (2019); Zhao et al. (2020a); Farahi et al. (2021a, 2021b, 2022); Wang et al. (2021a, 2022a)
PUNQ	Chang et al. (2015a, 2015b); Liu and Reynolds (2015), Liu et al. (2016a); Hutahaean et al. (2019)
Brugge	Chen et al. (2011, 2012); Zhao et al. (2019); Al-Aghbari et al. (2021, 2022a, 2022b)
Synthetic 2D model	Min et al. (2011); Yasari et al. (2013); Isebor and Durlofsky (2014a); Liu and Reynolds (2014, 2016b); Capolei et al. (2015); Christiansen et al. (2016, 2017); Moradi and Rasaei (2017); Bagherinezhad et al. (2017)
3D model (synthetic/ real)	Cardoso (2009); Hasan et al. (2013); Khan et al. (2013); Fonseca et al., (2014a; 2015); Safarzadeh et al. (2015); Chang et al. (2015b), Lu et al. (2017); Fu and Wen (2018); Hanea et al. (2019); Wang et al., (2021a, 2021c); Alpak et al. (2022)
No data	Harrison and Tweedie (1981); Xiao et al. (1998); Rahman et al. (2001); Bailey et al. (2005); Plaksina and Gildin (2015); Fu and Wen (2017a); Ji et al., (2017); Camara et al. (2018); Liu et al. (2021); Yan et al. (2021); Zhang et al. (2021)

reservoirs, they have not yet been applied in the context of multi-objective optimization for complex reservoirs (Meira et al., 2020; Fu et al., 2023; Hui et al., 2023).

5.2. Convergence challenges

Navigating the complex landscape of multi-objective optimization poses significant challenges, particularly concerning the convergence capabilities of existing algorithms. In the pursuit of simultaneously optimizing conflicting objectives, multi-objective algorithms face the challenging task of efficiently exploring the solution space to identify a diverse set of high-quality solutions. However, several factors may either decrease or increase the convergence capabilities of multi-objective algorithms for multi-objective production optimization. The list below summarizes the main aspects that may affect the performance of existing multi-objective approaches.

5.2.1. Allowed number of objective function evaluations

The allowed number of objective function evaluations plays a crucial role in determining the convergence behavior of a multi-objective algorithm. With a limited budget of objective function evaluations, the algorithm must make efficient use of each evaluation to explore and exploit the solution space effectively. A smaller number of evaluations may constrain the algorithm's ability to thoroughly explore the solution space, potentially leading to premature convergence to suboptimal regions or insufficient coverage of the Pareto front.

5.2.2. Stochasticity

Unlike deterministic algorithms, which follow a fixed set of rules to iteratively improve solutions, evolutionary algorithms introduce randomness into the optimization process through mechanisms such as mutation, crossover, and selection (Coello et al., 2007). This stochasticity allows evolutionary algorithms to explore diverse regions of the search space, facilitating the discovery of novel solutions and preventing premature convergence to local optima. By incorporating randomness, evolutionary algorithms can navigate non-convex search spaces where traditional optimization techniques may struggle (Schulze-Riegert et al., 2007). However, the stochastic nature of evolutionary algorithms also introduces challenges, such as convergence variability, as there is a need to run this type of algorithm multiple times to evaluate its average performance.

5.2.3. Pareto front geometry

The shape, complexity, and distribution of points along the Pareto front directly impact the convergence behavior of optimization algorithms. In scenarios where the Pareto front exhibits a smooth and convex shape, with well-separated solutions, convergence tends to be relatively straightforward as algorithms can efficiently explore and converge towards the front. However, in cases of non-convex, irregular, or discontinuous Pareto fronts with densely packed solutions, the convergence process becomes more challenging. Multi-objective algorithms may struggle to accurately locate and maintain a diverse set of non-dominated solutions, leading to premature convergence or the failure to adequately cover the Pareto front (Cheng et al., 2016).

5.2.4. Dimensionality

As the number of decision variables increases, the search space expands exponentially. This means that the optimizer needs to explore a larger number of practical solutions, making it increasingly difficult to find the optimal solutions within a reasonable amount of time. In addition, in high-dimensional spaces, data points become sparse, meaning that the available data becomes less representative of the entire search space (Coello et al., 2007). This can lead to difficulties in accurately estimating the objective function and constraints, impacting the optimization process.

6. Future directions

Historically, companies and the research community have preferred gradient-based methods over derivative-free approaches for multi-objective production optimization. Gradient-based methods have the advantages of being fast, mathematically accurate, and can be easily adapted for robust optimization (Hanea et al., 2019). However, gradient-based approaches have many disadvantages, such as not being able to find multiple non-dominated solutions, performing poorly in complex Pareto front geometries such as in non-convex, discontinuous, and non-smooth fronts, and being trapped into local optimum (Schulze-Riegert et al., 2007).

Recent research for multi-objective production optimization has been focused on a posteriori approach, using derivative-free methods, whose main benefits are: (i) they do not require manual identification of the optimal solutions; (ii) most of them achieve good results in many geometries of Pareto front and in non-linear optimization problems, (iii) and they may find several optimal solutions in a single optimization run. However, the proposed derivative-free methods are population-based meta-heuristics, which often require an infeasible number of reservoir simulations. This behavior is even worse in the presence of robust optimization, where each objective function evaluation is multiplied by the number of the ensemble of models. Without proper algorithm development, derivative-free methods may be impractical for real-world field development.

Therefore, in this section, we discuss future directions for multi-objective production optimization. These opportunities have been classified into four non-exclusive categories: (1) Improvements for derivative-free optimization, (2) Machine learning-based surrogate models, (3) Human interaction and knowledge incorporation, and (4) Robust optimization.

6.1. Improvements for derivative-free optimization

This section presents open or under-explored opportunities that researchers and practitioners from the petroleum industry can explore to increase convergence on derivative-free optimization by integrating intelligent mechanisms with statistically significant contributions to the field.

6.1.1. Diversity preservation

There are benchmark cases that use discrete decision variables. For instance, it is possible to consider a binary solution representation, where the decision vector contains several candidates well positions, and the binary solution representation represents the presence (1) or absence (0) of each candidates' well position (Avansi and Schiozer, 2015). In these problems, the optimization procedure becomes the exploration of different combinations of values, in which combinatorial optimization methods are required. However, combinatorial optimization methods may generate a large number of duplicated strategy vectors (Nojima et al., 2005). The existence of duplicated strategies results in inadequate diversity, which consequently slows the convergence speed, as more generations are required for a multi-objective algorithm to find the Pareto front. Although multi-objective algorithms have built-in methods to achieve diversity, such as environmental selection, this is mostly done in the objective space. To address this issue, researchers can use diversity preservation mechanisms to add diversity to the population and avoid duplicated strategies (Jiang et al., 2021). For instance, some examples of diversity preservation mechanisms include storing and prohibiting duplicated strategies (Glover, 1990), inserting random strategies into the population (Deb et al., 2007; Jiang and Yang, 2017), and estimating convergence direction (Gee et al., 2013). Currently, the only method applied for multi-objective production optimization that specifically addresses diversity preservation methods in the decision space is MOEA/D-NFTS (de Moraes and Coelho, 2022b). Further research in this area could improve the convergence capabilities and convergence speed of multi-objective algorithms when applied to multi-objective production optimization.

6.1.2. Global and local search

Multi-objective algorithms have the difficult task of exploring (global search) the often-large search space of solutions as much as possible while exploiting (local search) promising regions in detail. Although the former is usually done by every multi-objective algorithm, since they have inherent characteristics of global search, the latter is not naturally included in most algorithms. To handle both exploration and exploitation efficiently, multi-objective algorithms must be combined with local search methods. There are different local search methods in the literature that could be applied in the petroleum industry to enhance the exploitation capabilities of multi-objective algorithms, such as the Guided Local Search (Alhindi and Zhang, 2013), Simulated Annealing (Kirkpatrick et al., 1983), and Tabu Search (Glover, 1990), among others. There is also a class of methods known as Memetic Algorithms, which combine a baseline evolutionary algorithm with several local search methods, which have been shown to accelerate convergence and produce a larger and well-distributed Pareto front (Sun et al., 2020). However, as most local search methods perform exhaustive searches in the neighborhood of candidate solutions (which requires several evaluations of objective functions), they require adaptations (i.e., combination with surrogate models) before their implementation in multi-objective production optimization.

6.1.3. Adaptive hyper-parameters

Multi-objective algorithms are intrinsically parametrized. Mutation rates, crossover probabilities, tournament selection sizes, learning factors are some examples of parameters (also known as hyper-parameters) used in Pareto-based multi-objective algorithms such as NSGA-II and MOPSO. These hyper-parameters are used to control the optimization process (Eiben et al., 1999). Tuning these parameters is a difficult task in multi-objective production optimization due to the computationally expensive nature of reservoir simulation, as it increases the optimization time and may delay the project development. Regrettably, current state-of-the-art methods lack readily adjustable parameters. Hyper-parameter techniques have been proposed in the literature to minimize user input dependence.

Adaptive hyper-parameters use feedback from the optimization to automatically adapt the hyper-parameter values throughout the optimization (Eiben et al., 1999). The main idea is that fixed hyper-parameters may reduce the algorithm's performance, as these parameters may require different values in various stages of the optimization process (Thierens, 2002). Some examples of studies that investigated adaptive hyper-parameters were made by Thierens (2002), which provided an early proposition of some adaptive methods for the mutation rate of evolutionary algorithms. Zielinski and Laur (2007) proposed an adaptive scheme for the MOPSO algorithm to avoid user input parameters. Li et al. (2011) used adaptive methods to control crossover probabilities and scalarizing factors of the multi-objective differential evolution algorithm. More recently, Zhou et al. (2021) studied an adaptive hyper-parameter method based on Bayesian optimization for ship fuel consumption. These adaptive methods could inspire further research to reduce the requirement of user input parameters and even improve the performance of multi-objective algorithms in multi-objective production optimization.

6.1.4. Hyper-heuristics

Typically, multi-objective algorithms have a fixed heuristic structure (i.e., predefined crossover and mutation mechanisms) that is often overspecialized for a given problem. However, when the problem slightly changes, such methods tend to perform poorly and require manual modifications, a time-consuming task that delays the project development (Coello et al., 2020). Hyper-heuristics is a research field that has been promoted for multi-objective problems in the last decades to solve this issue.

Hyper-heuristics aim to identify the best combination of heuristics (such as crossover and mutation operators) to solve a particular problem, which may be very useful for multi-objective production optimization as there are several simulation models with different decision variables and Pareto front geometries that would, otherwise, require manual modifications to operate under such divergent scenarios (Burke et al., 2013). Hyper-heuristics have been combined with Pareto-based multi-objective algorithms to solve problems with combinatorial and continuous search spaces (see the survey by Burke et al. (2013)) and have been classified as one of the main research areas to be investigated in the future of multi-objective problems (Coello et al., 2020). However, hyper-heuristics increase the computational burden and should be considered alongside approximation mechanisms (such as surrogate models).

6.1.5. Scalable methods

In optimization theory, multi-objective problems contain two or three objective functions, while many-objective problems compose four objectives and above (Deb and Jain, 2014). Many-objective problems represent a whole different scenario for multi-objective production optimization. According to Deb and Jain (2014), the challenges of handling many-objective problems using traditional multi-objective algorithms include: (i) a large number of strategies is non-dominated; (ii) diversity evaluation becomes even more computationally expensive; (iii) recombination may be inefficient; (iv) representation of the trade-off surface is difficult; (v) performance metrics are computationally expensive; and (vi) visualization of the Pareto front is difficult. Considering that problems such the multi-objective production optimization have a wide range of potential objective functions, future research should concentrate efforts on scalable methods, that is, methods that can handle both multi and many-objective problems.

Decomposition-based methods are a class of scalable algorithms. Decomposition-based methods use a decomposition function that converts a multi-objective problem into scalar sub-problems to be optimized simultaneously (Zhang and Li, 2007). Examples of decomposition-based multi-objective algorithms include MOEA/D (Zhang and Li, 2007), NSGA-III (Deb and Jain, 2014), MOEA/D-RFTS (De Moraes and Coelho, 2022a), RVEA (Cheng et al., 2016), among others.

Currently, only a few works have used decomposition-based methods specifically developed for oil and gas field development (Zhao et al., 2020a; de Moraes and Coelho, 2022b). Decomposition-based methods are known to generate evenly spread set of solutions across the feasible region and are regarded as among the top algorithm designs for multi-objective optimization problems (Coello et al., 2020). However, it is important to note these algorithms cannot be used out-of-the-box for an arbitrary number of objectives because it becomes fatally necessary to include DM preferences to define specific regions of interest in the search space (the user preference articulation) in high-dimensional objective spaces. Nonetheless, decomposition-based methods can incorporate these user preference articulations, which makes the application of decomposition-based for multi-objective production optimization an interesting research topic.

6.1.6. Large-scale optimization

Multi-objective production optimization can encompass a large number of decision variables, possibly reaching several hundred. This circumstance affects the optimization process as the performance of multi-objective algorithms tends to deteriorate in high-dimensional decision spaces (Zille et al., 2018). Recent studies have been proposing different mechanisms to handle large-scale problems. Most of the works use the concept of cooperative co-evolution, where the idea is to optimize several independent populations, each one containing a subset of the original set of decision variables (Iorio and Li, 2004; Antonio and Coello Coello, 2013). Zille et al. (2018) use a framework based on problem transformation combined with grouping mechanisms. The authors tested their proposal on selected benchmarks with thousands of decision variables, and the results demonstrate an increase in the baseline multi-objective algorithm performance. These methods were experimented on test functions with inexpensive objective function calculations; therefore, applying cooperative co-evolution or problem transformation maybe difficult in the petroleum industry as they increase the necessary amount of function evaluations. However, results are promising and could be extended for further research on large-scale multi-objective production optimization.

6.1.7. Setting the initial population of strategies

One important aspect of multi-objective production optimization, which is often neglected, is the definition of the initial population, which contains the initial set of production strategies that will be used as input to the multi-objective algorithm, from which the optimization process will evolve. Most papers from the petroleum literature generate the initial population randomly. These random mechanisms have been questioned in the literature due to their insufficiency of diversity (Han et al., 2016; Liu et al., 2017; Gu and Wang, 2020). Considering that most multi-objective algorithms are highly sensible to the initial population, an inadequate mechanism to generate the initial population increases the time required to converge to the Pareto front. In computationally expensive problems, such as multi-objective production optimization, this represents a real threat. To overcome these problems, researchers have been proposing different approaches to create the initial population.

For instance, Friedrich and Wagner (2015) studied the effects of applying single objective algorithms methods to initially explore the search space and then use the obtained best solutions as the initial population of a multi-objective algorithm. The authors concluded that, for some problems (specially for expensive evaluations), there might be substantial improvements in quality and processing time.

Hamdan and Qudah (2015) investigated the performance of different sampling techniques, such as Latin hypercube and Quasi-random, as initialization methods to set the initial population of multi-objective algorithms. The authors also proposed a novel method called Quasi_LHS, which combined both the Latin hypercube with Quasi-random numbers. They assessed the mentioned methods using the NSGA-II algorithm with a set of continuous optimization problems and concluded that using the random initialization method does not perform well when compared to other methods.

Another technique to generate the initial population that has gained attention in the past years uses the Chaos theory, which studies the application of evolution functions that exhibit chaotic behaviors (Lu et al., 2013). Chaotic functions have been applied to multi-objective algorithms to induce chaotic behaviors, which helps the algorithms escape local optima, and to generate values for the decision variables that are indeed uniformly distributed, which have shown to be a more efficient initialization method on some problems (Yuan et al., 2002; Han et al., 2016; Liu et al., 2017; Gu and Wang, 2020).

Functions such as chaotic maps can be applied to multi-objective production optimization to generate values for the decision variables with a more uniform spread of points. With this approach, the initial strategies may be better distributed in the objective space, which can considerably increase the convergence process. Most chaotic procedures were designed for a continuous domain, but there are also strategies to generate points in discrete spaces, e.g., the Lambic Map (Lambić, 2015).

6.1.8. Performance metrics and statistical analysis

Comparing the performance of a multi-objective algorithm is not straightforward. Papers from the petroleum industry usually uses the Pareto front information, the number of generations it took to converge or specific petroleum-related information as performance metrics. Yet, there are other common and powerful performance metrics in optimization that could be used to provide further information about multi-objective algorithm performance. An example is the inverted generational distance (IGD), a convergence metric (Sierra and Coello Coello, 2005).

The IGD calculates the average Euclidean distance of each objective vector in the set of obtained solutions and the closest objective vector of the Pareto front. Eq. (5) defines the *IGD*:

$$IGD(PF,A) = \frac{1}{PF} \times \sum_{i=1}^{|PF|} \min_{j=1}^{A} d(PF_i, a_i) \tag{5}$$

where A is the set of obtained solutions (or the approximated front), PF is the Pareto front or a reference set, and PF_i is the *i-th* element of the Pareto front.

Although *IGD* is the most common performance metric in multi-objective optimization problems, there are other convergence metrics such as hyper volume (Zitzler and Thiele, 1999) and delta measure (Deb et al., 2002), coverage metrics like diversity measure (Deb et al., 2002), and performance success metrics like success counting (Sierra and Coello Coello, 2005), among others. For a list of performance metrics, please refer to the work by (Mirjalili and Lewis, 2015). Researchers from the petroleum industry can include performance metrics when comparing multiple algorithms to provide additional information about convergence, distribution, and coverage. After using performance metrics, statistical methods such as the non-parametric Wilcoxon signed-rank test (Wilcoxon, 1992) can be utilized to assert whether the contributions are statistically significant or not.

6.1.9. Hybrid methods

Another under explored approach in the literature of multi-objective production optimization is the combination of gradient-based with derivative-free algorithms as hybrid methods. The integration of different mechanisms is a common research topic in the optimization research field, which main goal is to create powerful methods that combines their advantages while reducing their individual drawbacks. Merging the mathematically accurate and fast gradient-based methods with Pareto-based algorithm's ability of automatically finding strategies that solves the problem could be also another interesting research topic.

6.2. Machine learning-based surrogate models

Production forecast requires complex simulations. These complex simulations involve solving nonlinear dynamic equations that represent the reservoir fluid behavior (Bertini et al., 2021). Due to this expensive nature, only some of the strategies can be properly evaluated using reservoir simulation (Golzari et al., 2015). Nonetheless, researchers have been employing alternatives to approximate the objective function values, using techniques known as fast objective function estimators, which includes surrogate models, proxies, streamline based simulators, among others (Ding et al., 2020).

Fast objective function estimators are often criticized due to their inability to comprehend the changes in a production strategy life cycle. For this matter, fast objective function estimators may not be a proper replacement for reservoir simulation since they may not produce equivalent results. However, if employed for derivative-free optimization, fast objective function estimators cannot be used as a replacement for conventional reservoir simulation but as a guide for the algorithm's internal mechanisms for the next best decision. Surrogate-assisted derivative-free methods have shown to have improved convergence capabilities when their genetic operators are combined with surrogate models to predict the best decision that could be made so far using the data collected during the optimization (de Moraes and Coelho, 2022a).

Alpak and Jain (2021) developed a machine-learning accelerated optimization method for well-location optimization (WLO) using support-vector regression (SVR) as a proxy to reduce computational costs. Field tests demonstrated a 40%–60% reduction in overall computational cost, validating the method's applicability for real-life WLO problems, though tuning SVR hyperparameters remains a challenge. Alpak et al. (2022) developed and validated BiMADS++, a novel parallel algorithm for bi-objective optimization in field-development planning, leveraging a new implementation of the mesh adaptive direct search (MADS) algorithm. Field tests on well-location optimization problems demonstrated significant computational efficiency, with BiMADS++ providing over fourfold speedup compared to traditional weighted-sum approaches while effectively identifying Pareto fronts without ad hoc parameter adjustments (Alpak, 2022). Moreover, Atadeger et al. (2023) investigate the use of deep learning-based (embed to control and observe (E2CO)) and kernel-based (LS-SVR) proxy models in nonlinearly constrained production optimization, comparing their computational efficiency and optimal results with high-fidelity simulators (HFS). The study finds that both proxy models achieve near-optimal NPV results with significantly reduced computational effort, and when enhanced with the iterative-sampling-refinement (ISR) technique, they closely match the NPV results obtained by HFS, demonstrating substantial efficiency in waterflooding scenarios.

This section discusses opportunities to approximate the objective function values of multi-objective optimization problems using machine learning-based surrogate models.

6.2.1. Random forest

Random forest is a statistical learning technique composed of an ensemble of decision trees, where each tree makes its own prediction of the objective functions of an unevaluated strategy, and the final result is the average result of all the decision tree predictions (Breiman, 2001). Fig. 7 illustrates this principle. This technique has gained attention over the past years as a surrogate model technique since it handles multiple objectives (Wang and Jin, 2018; Han and Wang, 2021; Rostamian et al., 2022; de Moraes and Coelho, 2022b). Random forests can handle both continuous and discrete decision variables, even nominal and categorical ones. In addition, since random forest has an if-then-else rule structure, when used with low-dimensional decision vectors, it is possible to plot the decision tree configuration to visually understand the decisions behind the prediction. These benefits make it a promising technique for data-driven production optimization surrogate modeling.

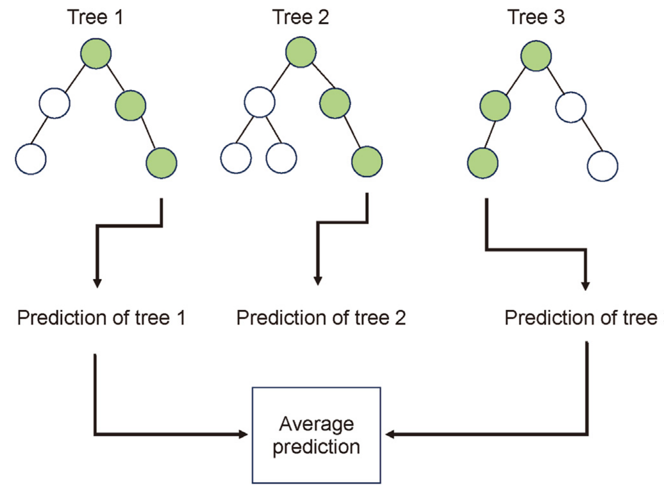

Fig. 7. Illustration of a random forest prediction with three decision trees.

6.2.2. Physics-informed

Surrogate models are generally data-driven (Zhao et al., 2020a, 2020b; Wang et al., 2021b). Data-driven surrogate models are trained using historical data (obtained by real reservoir simulation) to approximate the objective functions of unevaluated strategies (Li et al., 2020). The main benefit of data-driven models is that they are fast and simpler since they abstract the complexities of reservoir simulation (Bertini et al., 2021).

However, using data-driven surrogate models has its own drawbacks. Data acquisition is expensive in multi-objective optimization problems, especially in the presence of high-fidelity geological models (Golzari et al., 2015). Using data-driven models in such scenarios leads to a situation where DM must make decisions under partial (or incomplete) information. One conclusion that has been emerging in the literature is that modern surrogate models ignore a vast amount of prior knowledge of the problems they are being applied (Raissi et al., 2019).

Recent studies proposed physics-informed methods that incorporate knowledge from the laws of physics to solve complex problems. Instead of using data collected from expensive black-box models, these methods use physics formulations as “data”. Two prominent works are the ones by Raissi et al. (2019) and Sun et al. (2020), which incorporate deep neural network architectures to approximate values without simulation data. Deep neural architectures have shown an inherent ability to handle highly nonlinear and high-dimensional problems (Raissi et al., 2019; Liu et al., 2023). In their work, they show how these techniques can overcome some of the drawbacks of data-driven surrogate models and promote better convergence. Thus, using physics-based models for multi-objective optimization problems could be an important research topic in the future.

6.3. Human interaction and knowledge incorporation

Fully automatic methods can only find strategies that are optimal with regard of the simulation model, the decision variables, and its constraints, which may invariably be an over-simplified problem description (Scott et al., 2002). Therefore, complex problem such as production optimization benefits from human knowledge incorporation (Gaspar et al., 2016). This section presents ideas to allow human interaction and knowledge incorporation into the optimization process of multi-objective optimization problems.

6.3.1. Interactive methods

Interactive optimization or 'human-in-the-loop" methods are a class of algorithms that allow user interaction while they are running (Scott et al., 2002). The main benefit of having human interaction is that optimization can be more intelligent when managing constraints, settings, or even hyper-parameters that might be challenging to set at the beginning of the optimization (Brown, 2019). Scott et al. (2002) provided an overview of interactive methods. Since then, many methods have been developed. Some examples of interactive techniques that have been proposed in the literature include.

- Tezcaner and Köksalan (2011) propose an interactive algorithm for bi-objective problems based on linear utility functions. At every iteration, the algorithm asks the DMs to compare the two most efficient strategies that maximize or minimize both linear utility functions (one for each objective), to its adjacent strategies. The idea is to reduce the number of eligible candidates to efficient individuals and to guide the evolution using DMs knowledge.
- The visual interactive approach for stochastic multi-objective problems (VISMOP) (Balibek and Köksalan, 2012) is based on joint confidence regions, and it uses reference points, multivariate statistical analysis and the Tchebycheff decomposition. In VISMOP, the DM interactively decide which regions are more interesting to explore, through the definition of both the direction and the step size to that direction.
- In the interactive method using reservation and aspiration levels for evolutionary multi-objective optimization (IRA-EMO) algorithm (Saborido et al., 2019), DMs help the algorithm by indicating their preferences at every iteration, in the form of aspiration (lower) and reservation (upper) points in the objective space. The DMs can also interactively set the number of solutions they want to analyze at every iteration.
- The interactive centroid method (ICM) (Chou et al., 2020) is an interactive method, originally designed for multi-objective dispatching problems, that allows DMs to choose, at every iteration the solutions that will be categorized into the following three categories: (a) the most preferred; (b) the least preferred; and (c) indecisive.

Besides, an engineering research field where interactive methods have been studied and which may also inspire future projects in the petroleum field is architectural and structural design (Brown, 2019). In this field, architects and engineers use interactive methods such as paraGEN (Turrin et al., 2011), a framework that uses genetic algorithm structures (such as selection, recombination and mutation) to build cooperative systems (human-computer interactions) to explore the alternatives for designing buildings and other structures, and structure FIT (Mueller and Ochsendorf, 2015), a tool with a live graphical user interface where experts can interactively modify strategies returned by an optimization algorithm.

The concept of interactive methods can be applied to multi-objective production optimization. Allowing human interaction gives an opportunity for researchers and petroleum engineers to interactively modify solutions according to the institution or company goals. Besides, it is possible to accelerate the optimization process by modifying the variables to desired configurations that are well-established concepts in the petroleum field but that is difficult to constraint and/or would require many iterations of a multi-objective algorithm to achieve (e.g., move injector wells to high-permeability zones and move producer wells to high oil saturation zones). This mechanism may introduce individuals in the multi-objective algorithm's population that accelerate the optimization process.

The development and implementation of interactive methods is a multidisciplinary task since it requires professionals from optimization, petroleum engineering and information visualization research fields. Besides, determining the most appropriate division between human and computer labor is one of the most important questions in interactive methods, which may require a considerably effort (Scott et al., 2002). However, future research in interactive methods could lead to the development of cooperative intelligent systems for multi-objective production optimization.

6.3.2. Multi-criteria decision-making

The number of solutions in a multi-objective production optimization is often large. One question that arises after the optimization takes place is, "How to select the best alternative among these several conceptually optimal strategies?". Multi-criteria decision-making (MCDM) methods are a class of decision-making methods that could assist the petroleum industry. The purpose of MCDM is to sort or classify solutions according to DMs' preferences, to help them decide on a single alternative when there is a set of optimal possibilities and a choice between them must be made. The field of MCDM has addressed and developed powerful methods over the last decades to help DMs decide on solutions to complex multi-objective problems (Miettinen, 1998).

More recently, researchers have proposed the hybridization of multi-objective algorithms and MCDM techniques to incorporate DMs preferences into the optimization process. These hybrid methods can be classified into three main categories: before, when DMs preferences are incorporated before the optimization takes place; during, when the methods incorporate DMs preferences interactively (as shown in Section 5.3.1); and after, when DMs preferences are incorporated after the search (Purshouse et al., 2014). The hybrid multi-objective algorithm that uses MCDM techniques has been shown to be particularly effective in complex problems when DMs knowledge can be modeled as user preferences (Marler and Arora, 2004). Although the selection for a production strategy is a multi-step process involving several analytical observations, the decision analysis could be highly supported by the application of an MCDM technique.

6.4. Robust optimization

Robust optimization aims to find strategies that remain slightly unchanged when presented with uncertain conditions (Beyer and Sendhoff, 2007). This section presents a class of algorithms, in the context of multi-objective optimization, to handle time-varying features. It also introduces the concept of representative models.

6.4.1. Dynamic methods

A dynamic optimization problem is when objective functions, constraints, or decision variable boundaries change over time (Deb et al., 2007). There are two computational procedures to handle dynamic problems: (1) generating a handful of events to be solved off-line and (2) optimizing the time-varying features on-line (Deb et al., 2007). Robust optimization for multi-objective production optimization has been performed using approach (1), but efforts have been made by the optimization scientific community to develop methods to handle the second procedure. Dynamic multi-objective optimization methods (DMOM) have been developed to handle time-varying features in multi-objective problems using several propositions.

There are DMOMs that introduce random or mutated solutions whenever a change is detected (Deb et al., 2007), keep previous solutions, and use prediction methods to generate novel solutions according to the movement of change (Jiang and Yang, 2017), keep records of previous solutions to be possible to change back to previous steps (Azzouz et al., 2017), use co-evolving populations to optimize solutions in different spaces (Branke et al., 2000), and combine the multi-objective algorithms with regression-based prediction methods such as Kalman filter to track the dynamic changes (Muruganantham et al., 2016), among others.

A DMOM would have little to no effect under geological uncertainties. However, there are some other uncertain sources, such as economic, that could be studied as a DMOM. As the time-value of money occurs outside of a reservoir, there would be no need to build different simulation models to compute, for instance, different crude oil prices. Instead, the problem could be defined as a Dynamic multi-objective optimization problem (DMOP) such as demonstrated in Eq. (6):

$$\min F(\vec{x}) = \langle f_1(\vec{x}, t), f_2(\vec{x}, t), \ldots, f_M(\vec{x}, t) \rangle \quad (6)$$

$$\text{s.t. } \vec{x} \in \Omega, t \in \Omega t$$

where Ω is the decision feasible region, t is a discrete-time step, $\vec{x} = [x_1, x_2, \ldots, x_N]$ is the strategy vector, and Ωt is the time step feasible region. In this case, the objective functions are subject to change over time due to economic uncertainties. Instead of performing an off-line procedure to handle it, an on-line approach could be made using a DMOM. DMOM is still a fairly recent research topic, but it opens an opportunity to develop dynamic methods for multi-objective problems that could expand the possibilities of optimization under uncertainties in some cases.

6.4.2. Representative models

There are many uncertainty sources in an oil and gas field development, including geological, economic, technological, and political uncertainties, among others (Schiozer et al., 2015). Ideally, each one of these uncertainties should be modeled and considered during the optimization. However, this would be infeasible as the number of simulations would be impractically time-consuming. For this purpose, the identification of representative models (RM) has emerged in the literature, whose central idea is the identification and selection of a subset of models with the highest impact on the decision-making process (Schiozer et al., 2004). Sarma et al. (2013) proposed a minimax approach for selecting a few statistically representative reservoir models from a large ensemble, efficiently matching target percentiles of multiple output responses while ensuring maximal difference in the input uncertainty space. This approach demonstrated superior performance and speed compared to traditional clustering methods, facilitating better decision-making and planning in the face of large model sets. Optimization procedures, such as RMFinder, have been developed as scenario reduction techniques (Meira et al., 2020). According to the authors, RMFinder selects RMs that effectively decrease the number of simulation models while maintaining the representativeness of the problem's uncertainties. Scenario reduction techniques such as RMFinder can severely decrease the number of reservoir simulations in robust production optimization problems and should be considered as an important research topic for the future (Meira et al., 2020). Gao et al. (2023) proposed a two-stage multi-objective optimization strategy to select representative deterministic models (P10, P50, P90) in uncertainty quantification workflows, balancing conflicting requirements and constraints efficiently. This method, which is evaluated on realistic examples, demonstrates robustness and efficiency, significantly reducing computational costs while ensuring high-quality model selection that meets both performance and regulatory constraints.

7. Conclusions

This paper has presented an essential review of multi-objective production optimization. Basic concepts and existing work in the literature were discussed. This review shows that there have been significant advancements in the past decades, especially for derivative-free methods. A posteriori approaches, mainly composed of Pareto-based methods have been developed to find production strategies that automatically represent the best trade-offs between multiple conflicting criteria.

However, as the developed derivative-free methods for multi-objective production optimization are population-based meta-heuristics, they often require an infeasible number of reservoir simulations. This makes them impractical for some real-world problems, especially in ensemble-based field development optimization. Furthermore, several studies show that methods like stochastic simplex approximate gradient (StoSAG) demonstrate computational efficiency when dealing with continuous optimization parameters and multi-objective functions. Therefore, this paper also presents open challenges and future directions to build efficient algorithms, organized into four distinct categories: improvements for Pareto-based optimization, machine learning-based surrogate models, human interaction and knowledge incorporation, and robust optimization.

Code availability section

The paper is purely theoretical and does not involve any implementation or source code. Therefore, no code is available for this study.

CRediT authorship contribution statement

Auref Rostamian: Writing – review & editing, Writing – original draft, Methodology, Formal analysis, Data curation, Conceptualization. **Matheus Bernardelli de Moraes:** Writing – review & editing, Writing – original draft, Methodology, Formal analysis, Data curation, Conceptualization. **Denis José Schiozer:** Writing – review & editing, Validation, Supervision, Project administration, Funding acquisition, Conceptualization. **Guilherme Palermo Coelho:** Writing – original draft, Supervision, Project administration, Funding acquisition, Conceptualization.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Acknowledgments

We gratefully acknowledge the support of EPIC – Energy Production Innovation Center, hosted by the University of Campinas (UNICAMP) and sponsored by Equinor Brazil and FAPESP – São Paulo Research Foundation (2021/04878- 7 and 2017/15736-3). We acknowledge the support of ANP (Brazil's National Oil, Natural Gas and Biofuels Agency) through the R&D levy regulation. Acknowledgements are extended to the Center for Petroleum Studies (CEPETRO), School of Mechanical Engineering (FEM) and the School of Technology (FT). This study was financed in part by the

Coordenação de Aperfeiçoamento de Pessoal de Nível Superior - Brasil (CAPES) – Financing Code 001.

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