Pet.Sci.(2012)9:408-415
DOI 10.1007/s12182-012-0225-6

Production sharing contract: An analysis based on an oil price stochastic process

Liu Mingming[1], Wang Zhen[1*], Zhao Lin[2], Pan Yanni[1] and Xiao Fei[1]

[1] School of Business Administration, China University of Petroleum, Beijing 102249, China
[2] China National Oil & Gas Exploration and Development Corporation, Beijing 100083, China

Abstract: Assuming that oil price follows the stochastic processes of Geometric Brownian Motion (GBM) or the Mean-Reverting Process (MRP), this paper takes the net present value (NPV) as an economic index and models the PSC in 11 different scenarios by changing the value of each contract element (i.e. royalty, cost oil, profit oil as well as income tax). Then the NPVs are shown in probability density graphs to investigate the effect of different elements on contract economics. The results show that under oil price uncertainty the influence of profit oil and income tax on NPV are more significant than those of royalty and cost oil, while a tax holiday could improve the contractor's financial status remarkably. Results also show that MRP is more appropriate for cases with low future oil price volatility, and GBM is best for high future oil price volatility.

Key words: Production sharing, Geometric Brownian motion, Mean-Reverting Process, oil contract, international petroleum cooperation

1 Instruction

Oil contracts are always regarded as "the Bible" in international cooperation for being the foundation of oil companies' operation in overseas oil & gas exploration and development. A common type, production sharing contracts (PSCs) have features that the oil resources are owned by the host government (often represented by national oil companies, NOCs), while foreign oil companies (FOCs) undertake all risks as well as the cost in the entire exploration process, then production is split at an agreed rate between the NOC and FOC. Also, FOCs have to pay income tax on their share of profit oil (Johnston, 2003; Ge et al, 2004).

Many Chinese and overseas scholars have put great efforts into the study of PSC. Johnston (1994) presented and investigated the contract elements of PSC in detail in *International Petroleum Fiscal Systems and Production Sharing Contracts*. Wu and Wang (2006) studied the risks in PSC. Mudford and Stegemeier (2003) examined the sensitivity of production sharing contract terms under both technical and price uncertainties; also, they compared different PSCs in three countries, Egypt, Angola and Equatorial Guinea. Bindemann (1999) investigated the economics of PSC at a fixed oil price and the impacts of contract elements on PSC economic indices, NPV and IRR. Wang et al (2010b) modeled the impact of contract elements on PSC economics by assuming oil price follows Geometric Brownian Motion. Hao and Kaiser (2010) constructed a meta model for modeling China's offshore production sharing contracts by using a probabilistic approach.

The oil industry's features of large investment, long period and high risk, as well as the particularities of international petroleum cooperation modes, oil price and major contract elements both play a central role for evaluating projects (Luo and Yan, 2010). The future oil price is full of uncertainties, and due to oil's more and more significant financial attributes, the fluctuation track of oil price is more like a stochastic process (Chen et al, 2008; Lin and Liu, 2008). This paper is to construct an economic analysis framework considering future fluctuations of oil price. Taking the Geometric Brownian Motion and Mean-Reverting Process separately as oil price stochastic processes, the paper investigates contract economics based on the probability density graphs of NPVs from simulations. They both better describe future uncertainties and help making results more reasonable and more persuasive. In addition, the impacts of different contract elements on PSC economics are studied. We hope the findings of this paper will contribute a new reference for bidding, negotiating and decision making to the business world

2 PSC contract elements

In general, there are two key participators in a PSC, a foreign oil company (FOC) and a government representative which is often a national oil company (NOC). Once oil is produced, the FOC has to pay some royalty. After FOC recovers its costs at a pre-specified percentage of production,

*Corresponding author. email: wzhen_2000@yahoo.cn
Received December 6, 2011

the so-called cost oil, the remainder of production (profit oil) is then split between NOC and FOC at an agreed rate. Meanwhile, an income tax can commonly be imposed on FOC for its operations on the territory and has to be paid from its share of profit oil. Therefore, main contract elements of PSCs include royalty, cost oil, profit oil and income tax (Bindemann, 1999).

2.1 Royalty

Royalty is one of the most basic elements for a contract of international oil cooperation. Once the production starts, FOCs have to pay royalty (by oil or cash) to host country government. In reality, royalty is not always set as a fixed share of oil production, and can be adjusted according to average daily output. Generally it varies from 6% of total production to 15%, as too high a proportion could exert negative effect on FOCs and impede production cooperation.

2.2 Cost recovery

Before the initiation of production sharing, PSC allows contractors (FOCs) to recover their costs on exploration, development of oil fields and operation at a pre-specified percentage of production, the so-called cost oil. Most PSCs have a cost-oil limit of say 60 percent of production, while allowing unrecovered costs to be carried forward and recovered in the next period.

2.3 Profit oil

After subtraction of royalty and cost oil, the remainder of the production, the so-called profit oil, is then split between FOC and NOC at an agreed rate, like 40/60.

2.4 Income tax

When FOCs get their share of profit oil, they have to pay income tax at an agreed rate stipulated in contracts or local laws. Sometimes, the host government would set a tax holiday of several years, so as to encourage FOCs to carry out exploration and development on its territory.

3 Simulation framework for PSC

3.1 Selection of oil price stochastic process

In the international petroleum market, besides the basic factors of supply and demand, there are many other factors such as geopolitical perceptions and speculation activities that act upon oil prices. These factors are complicated and intertwined, and difficult to quantify, making the track of oil price movement more like a stochastic process and unpredictable. In this study, Geometric Brownian Motion (GBM) and Mean-Reverting Process (MRP) are selected to describe the movement of oil prices for economics analysis (Wang et al, 2010a, 2010b; Schwartz, 1997; Wang and Li, 2010).

3.1.1 Geometric Brownian motion

If the price of an underlying asset follows a Geometric Brownian Motion, it will show that:

$$\mathrm{d}S = \mu S\mathrm{d}t + \sigma S\mathrm{d}z \tag{1}$$

where S is the price of underlying asset, and variables μ and σ are respectively the expected proportional growth rate and the volatility in the underlying asset price, while dz is a Wiener process, $\mathrm{d}z = \varepsilon\sqrt{\mathrm{d}t}, \varepsilon \sim N(0,1)$.

3.1.2 Mean-Reverting Process

A Mean-Reverting Process refers to the process where prices will revert to their mean value when they are at too high or too low a level. If the price of an underlying asset follows a Mean-Reverting Process, it can be expressed as:

$$\mathrm{d}S = \eta S(\bar{S} - S)\mathrm{d}t + \sigma S\mathrm{d}z \tag{2}$$

where $\bar{S}$ is the long term average value to which the underlying asset price tends to revert and η is the mean reversion rate. Equation (2) shows that being affected by a certain restoring force, the price of underlying asset moves towards the long-term average price.

3.2 Realization of oil price stochastic process

3.2.1 Realization of oil price following Geometric Brownian motion

Letting x=ln$S$ and using $It\hat{o}'s$ Lemma, we have (Hull, 2000):

$$\mathrm{d}x = \mathrm{d}(\ln S) = (\mu - \frac{\sigma^2}{2})\mathrm{d}t + \sigma\mathrm{d}z \tag{3}$$

Equation (3) implies that in a time period of t to T, ln$S$ follows a normal distribution:

$$\ln S_T - \ln S_t = \phi[(\mu - \frac{\sigma^2}{2})(T-t), \sigma\sqrt{T-t}] \tag{4}$$

where $\phi(m,s)$ denotes a normal distribution with mean m and standard deviation s. Assuming risk neutrality, replace the expected proportional growth rate μ with risk-free rate of interest r, then equation (4) will change into that:

$$\ln S_T - \ln S_t = \phi[(r - \frac{\sigma^2}{2})(T-t), \sigma\sqrt{T-t}] \tag{5}$$

From equation (5), we know that the spot price S_T at a future time T follows a lognormal distribution.

With some simple conversions to equation (5), spot price at time T will be expressed as:

$$S_T = S_t \cdot \exp[(r - \frac{\sigma^2}{2})(T-t) + \sigma\sqrt{T-t}\cdot\varepsilon] \tag{6}$$

where $\varepsilon \sim N(0,1)$.

Defining t=0, the equation (6) will then be:

$$S_T = S_0 \cdot \exp[(r - \frac{\sigma^2}{2})\cdot T + \sigma\sqrt{T}\cdot\varepsilon] \tag{7}$$

Equation (7) tells that in order to determine oil price S_T at time T, the following four variables need to be defined: the initial price S_0, risk-free interest rate r, the oil price volatility σ, as well as random number ε which follows normal distribution.

(1) Risk-free interest rate (*r*)

The average value of "*Daily Treasury Real Long-Term Rates*" in 2011, i.e.1.19%, published by US Department of

Treasury, is taken as the basis of risk-free interest rate r, together with a recommended market risk premium of 2%, the risk-free interest rate in this work is set at 3.19%.

(2) Volatility of oil price

In this work, the oil price volatility was calculated by using the equation proposed by Davis (1998) as follows:

$$\sigma = \frac{1}{\sqrt{\tau}}\sqrt{\frac{1}{n-1}\sum_{i=1}^{n}\left(u_i - \bar{u}\right)^2} \tag{8}$$

where, $u_i = \ln\frac{S_i}{S_{i-1}}$, S_i is the crude oil price at the end of time interval i; τ is the time interval span of the historical oil price data, counted in years.

Based on 26 years of oil price data from the US Energy Information Administration (EIA), we use 145 monthly data of oil price from 01/2000 to 01/2012, and get oil price volatility σ=0.309.

(3) Random number ε

The ε in equation (7) follows normal distribution. Here software @Risk is applied to generate random numbers for oil price calculation.

Finally, r, σ and ε values are substituted in equation (7), and the oil price series following Geometric Brownian Motion is simulated.

3.2.2 Realization of oil price following mean-reverting process

Supposing that oil price S follows a Mean-Reverting Process (Schwartz, 1997):

$$\mathrm{d}S = \eta S(m - \ln S)\mathrm{d}t + \sigma S\mathrm{d}z \tag{9}$$

where m is the logarithm of long-run mean value to which the oil price tends to revert.

Letting $x = \ln S$ and using $It\hat{o}'s$ Lemma, we have:

$$\mathrm{d}x = \eta^* \cdot \left(m^* - x\right)\mathrm{d}t + \sigma\mathrm{d}z \tag{10}$$

where $\eta^* = \eta$, $m^* = m - \sigma^2/(2\eta)$.

Considering that x has a normal distribution, its expected value and variance are as follows:

$$\begin{aligned} E(x(t)) &= m^* + (x(0) - m^*)\exp(-\eta t) \\ &= x(0)\exp(-\eta t) + m^*(1 - \exp(-\eta t)) \end{aligned} \tag{11}$$

$$VAR(x(t)) = \left[1 - \exp(-2\eta t)\right]\sigma^2/(2\eta) \tag{12}$$

The equation (10) is a continuous time version of the first-order autoregressive process, AR(1). It is the limiting case of the AR(1) process (Dixit & Pindyck,1994; Schwartz, 1997):

$$x_t - x_{t-1} = m^*(1 - e^{-\eta^*\Delta t}) + (e^{-\eta^*\Delta t} - 1)x_{t-1} + \varepsilon_t \tag{13}$$

where ε_t is normally distributed with zero mean and standard deviation σ_0 and $\sigma_\varepsilon^2 = [1 - \exp(-2\eta)]\sigma^2/(2\eta)$.

Running the regression:

$$x_t - x_{t-1} = a + bx_{t-1} + \varepsilon_t \tag{14}$$

Then the value of the parameters are:

$m^* = -a/b$;

$\eta^* = -\ln(1+b)$;

$\sigma = \sigma_\varepsilon\sqrt{2\ln(1+b)/\left((1+b)^2 - 1\right)}$

Based on the monthly West Texas Intermediate (WTI) crude oil prices from 01/2000 to 01/2012 from EIA, the long-run mean value to which oil price tends to revert was calculated to be 103.32 \$/b, with the oil price volatility σ=0.29 and the mean-reverting rate η=0.1381.

Since the variables σ and η are determined, with equations (11) and (12), the mean value and variance of normal distribution for logarithm oil prices at year t can be obtained. Here t, varying from 1 to 15, corresponds to year 2012 to 2026, and using Monte Carlo simulation, the oil prices that follow Mean-Reverting Process can also be obtained.

3.3 Sample size for Monte Carlo Simulation

In Monte Carlo simulation, the oil price is randomly selected. In this process, sample size is very important. If the sample size is too small, it cannot reflect the main features of oil prices, thus the results cannot be used as the basis for decision-making. If the sample size is too large, the simulation cost will increase. The methods used for determining sample size in Monte Carlo simulation include the absolute error method and the relative error method (Zhang and Wang, 2004). In this work, the relative error method is used for oil price simulation, i.e. the relative error, computed by the difference between sample mean and population mean divided by the later, should fall into a small scope at a certain probability.

Taking the Mean-Reverting Process as an example. The changes of mean and variance of oil price samples with sample size are shown in Fig. 1.

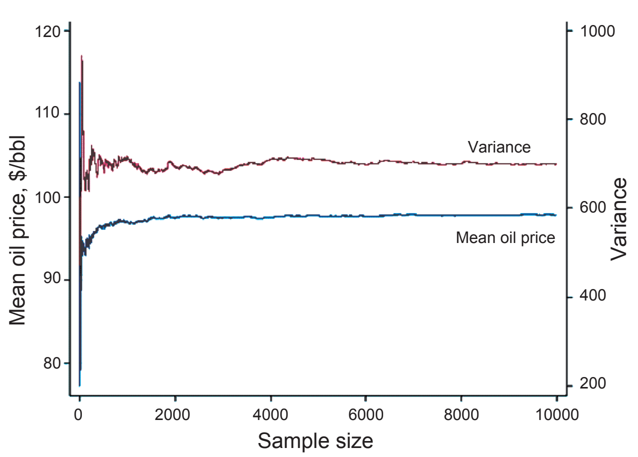

Fig. 1 Changes of mean value and variance of oil prices with sample size

As Fig. 1 shows, when the sample size is small, the mean value and variance of oil prices oscillate greatly. But the mean value and variance tends to stable when the sample size is larger than a certain scale. According to the relative error method, the relative error coefficient is assumed to be 0.05, and the computing results show that once the sample size reaches 9,800, the probability of the relative error less than 0.05 will achieve 0.99. Therefore, 10,000 is set as the sample size for Monte Carlo simulation, and for Geometric Brownian Motion as well since their results are close.

3.4 Scenarios design

3.4.1 Case introduction

Taking a 15-year PSC starting from 2012 as example, the period includes 3 years of exploration and 12 years of production phase. The preliminary agreement specifies that the royalty is 10%, and the limit for cost recovery is 40% of oil production; the profit oil is split at 60/40 (Government (60), Contractor (40)) and the income tax rate is set as 30%.

3.4.2 Scenarios design

The preliminary agreement will be used as the original assumption. Besides, the value sets of contract elements for scenario analysis are as follows:

(1) Royalty

So far the rate of royalty is often seen between 6% and 15%, so three royalty scenarios are stipulated as 6%, 10% and 15%.

(2) Cost oil

Before profit oil is split, the FOC can recover its exploration, development and operation costs from gross production, but usually there is a cost-limit. We set four cost oil scenarios: 0%, 20%, 40% and 60%.

(3) Splitting ratio of profit oil

Profit oil is split between the government representative (or NOC) and the contractor (FOC) at an agreed rate. Three scenarios (NOC/FOC) are assumed for stimulation: 60/40, 50/50, 40/60.

(4) Income tax

Income tax is generally imposed. But in order to attract investments, some countries would set a tax holiday to encourage FOCs, and some countries even simply set the whole period tax-free. Accordingly, four income tax scenarios are drafted: 20%, 30%, 40% and 30% with a 5-year tax holiday.

4 Simulation results

By using software @Risk, the 10,000 oil price paths are generated and used respectively to calculate NPV based on the contract; Then, the probability density of NPVs are calculated and the corresponding graphs are given.

4.1 PSC simulation result

The statistic results of NPVs for the original assumption under the two oil price stochastic processes are shown in Figs. 2 and 3.

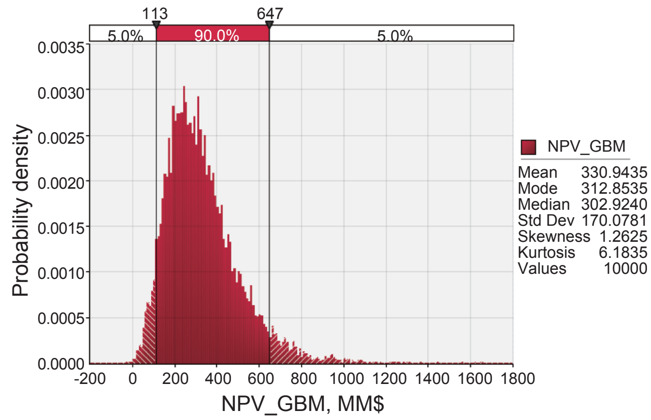

Fig. 2 Probability density graphs of NPVs under GBM

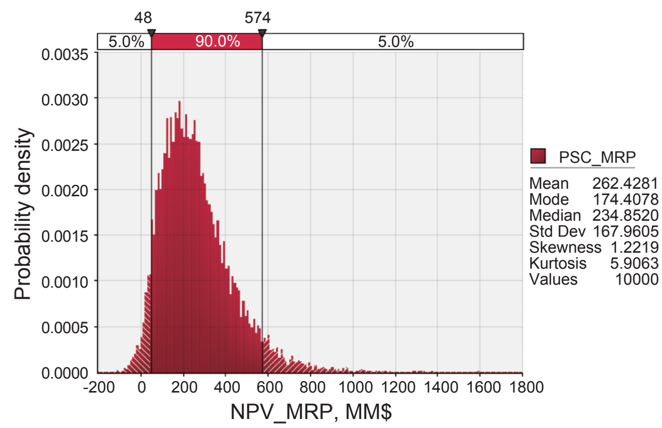

Fig. 3 Probability density graphs of NPVs under MRP

The simulation results in Figs. 2 and 3 show that the proposed contract has quite a high profit potential, although there are some differences under GBM and MRP. For GBM, NPVs mainly fall into the range of 100–500 with a frequency of 0.8102, making its mean as high as 330.9 million dollar. For MRP, NPVs principally fall into the range of 0–400 with a little higher frequency of 0.8178, thus its mean is lower than GBM's. Through a further comparison of their distributions' characteristics, the GBM has a little higher volatility as well as a higher skewness and kurtosis than those of MRP's, indicating that the oil price following GBM has a higher volatility, which is consistent with the simulation results of oil price.

4.2 Effects of different contract elements on contract economics

4.2.1 Effect of royalty

The results of contract economics simulation for different royalty scenarios are shown in Fig. 4. It can be seen that the probability density of NPVs does not change greatly with an increase of royalty. When the royalty increases from 6% to 10% and then to 15%, NPV respectively falls into the ranges of 127–684, 113–647 and 97–600 under GBM with a probability of 90%, and 59–609, 48–574 and 35–532 respectively under MRP. The mean NPV is 354.3, 330.9 and 301.8 million dollars under GBM, and 282.7, 262.4 and 237.1 million dollars under MRP, respectively. Both the probability density and mean NPV indicate that the contract would result in a better return if oil price follows a GBM stochastic process.

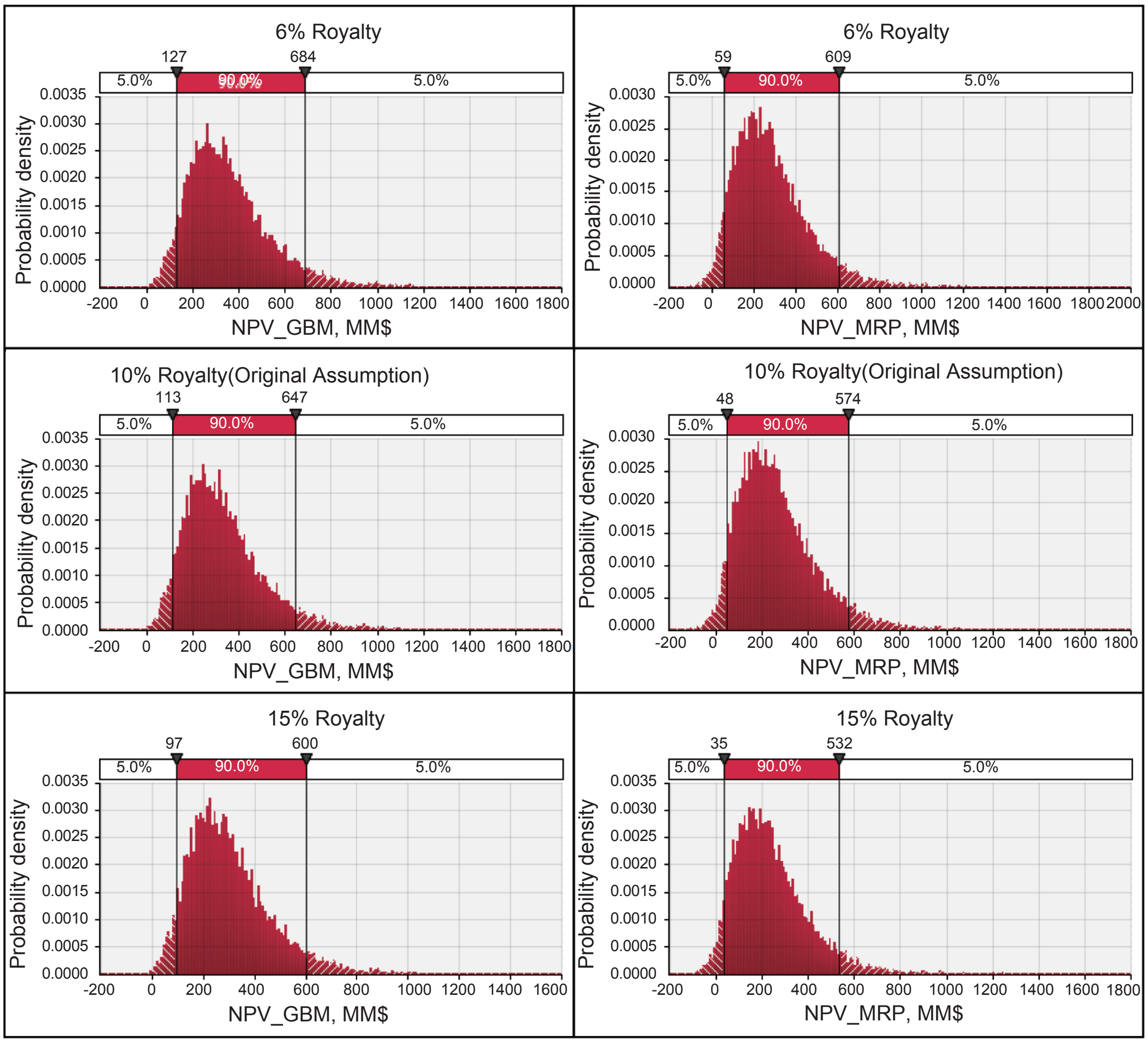

Fig. 4 Probability density graphs of NPVs at different Royalties (left is under GBM and right under MRP)

4.2.2 Effect of cost oil

Four scenarios with various limits of cost oil are designed in this paper: 0%, 20%, 40% and 60%, where 0% means no cost oil at all. Simulation results shown in Fig. 5 indicate that the effect of cost oil on NPV is significant, especially under MRP. The existence of cost oil in contract significantly improves the NPV for the contractor. When the cost oil rate is low and unfortunately the oil price also stays on low level, there is a risk that the amortized cost might not be recovered entirely, thus makes NPV negative, for example, the scenario with 0% cost oil rate under MRP, in which the contractor can get a negative NPV at a high probability around 48%. However, when the cost oil rate is increased to reach a certain level, at which most of the amortized cost can be recovered, further increase of cost oil rate would not make any noticeable difference in NPV, such as from 40% to 60% in this paper.

4.2.3 Effect of profit oil

Three scenarios are designed to investigate the effect of profit oil rate (government/contractor), and the results shown in Fig. 6 show that the effect of profit oil on NPV is significant under both GBM and MRP.

When the profit oil rate (government/contractor) changes from 60/40 to 50/50 and then to 40/60, NPV will fall into (113, 647), (169, 835) and (224, 1023) with a probability of 90% respectively under GBM, and these intervals will change into (48, 574), (87, 744) and (127, 914) under MRP. The mean value of NPV increases from 330.9 to 440.3 and to 549.6 million dollars under GBM, and the corresponding values under MRP are 262.4, 354.3 and 446.1 million dollars. Therefore, a higher profit oil rate allocated to a contractor would lead to a higher probability of gaining high NPV, and as well as higher resistance against the risk of low oil prices.

4.2.4 Effect of income tax

Once the profit oil is split, income tax would be levied from the Contractor. Sometimes, the host country would give FOCs a tax holiday, e.g. 5 years, to encourage investment. As stipulated above, four scenarios of income tax respectively at 20%, 30%, 40% and 30% with a 5-year tax holiday are designed, and the simulation results are shown in Fig. 7.

Fig. 7 shows that the effect of income tax rate on the probability distribution of NPVs is also significant when the oil price is uncertain. When the income tax rate increases from 20% to 30% and then to 40%, NPV mean value under GBM decreases from 393.4 to 330.9 and down to 268.5

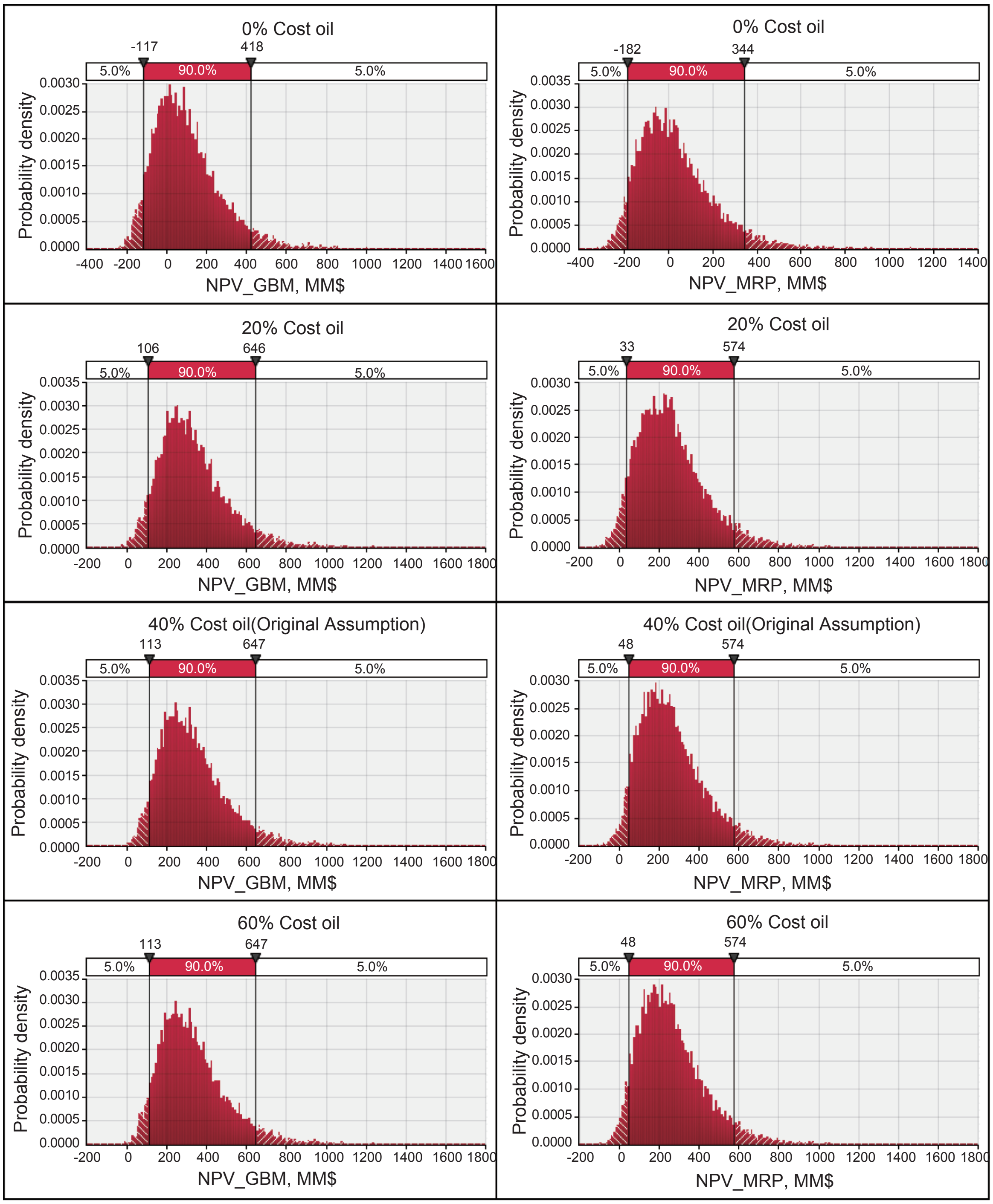

Fig. 5 Probability density graphs of NPVs at different cost oil rates (left is under GBM and right under MRP)

million dollars, while under MRP, the corresponding NPV mean values are 314.9, 262.4 and 209.9 million dollars. The simulation results reveals that a tax holiday would truly improve NPV (in this paper, NPV with 5-year tax holiday is superior to that with a tax of 20%), and is attractive to FOCs. Meanwhile, by comparing the results, we find that PSC's economics are more sensitive to tax under MRP but NPV mean value changes in a larger range.

5 Conclusions

Geometric Brownian Motion (GBM) and Mean-Reverting Process (MRP) were taken into account to model the uncertain oil price, and probability density graph was used to reveal the simulation results of contractor's NPV with respect to different contract elements. The results indicate that the profit oil has the most significant influence on NPV followed by income tax, royalty and cost oil. Low proportion of cost

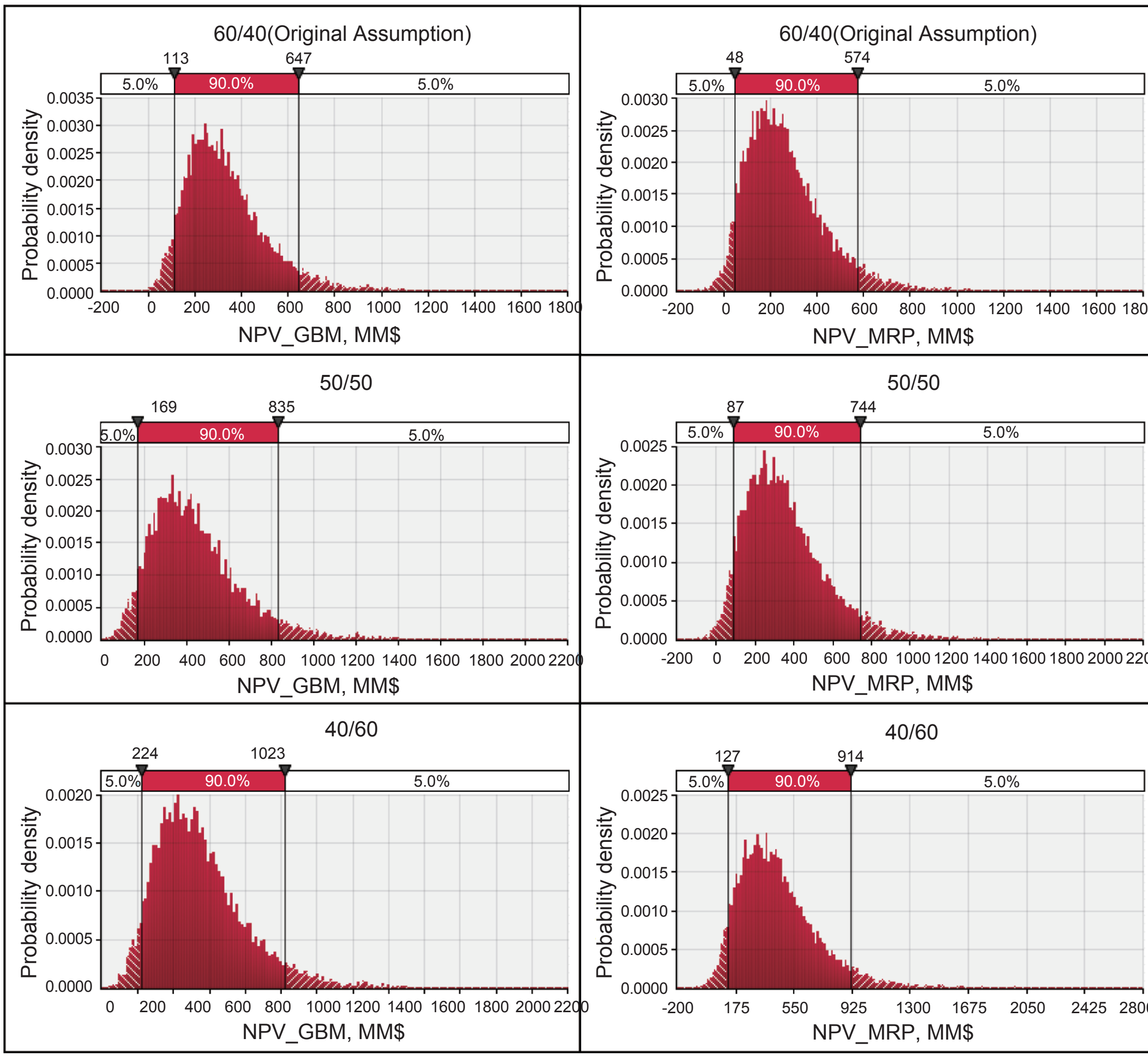

Fig. 6 Probability density graphs of NPVs at different profit oil rates (left is under GBM and right under MRP)

oil could result in contractor's weakness against low oil price, while a tax holiday would increase the contractor's NPV substantially. As to the extent of NPV changes with oil price, MRP gets higher results than that of GBM, but with smaller scale and NPV range. Also, the study finds MRP is more used in cases that have lower expectation of oil price volatility, and GBM is favored where expectations of oil price volatility are high.

Acknowledgements

The authors are grateful for financial support from Key Projects of Philosophy and Social Sciences Research of Ministry of Education (09JZD0038).

References

Bindemann K. Production-Sharing Agreements: An Economic Analysis. Oxford: Oxford Institute for Energy Studies, 1999

Chen H T, Zhou D Q, Wang Q W. Review of oil finance theory. Economic Perspectives. 2008. 7: 99-105 (in Chinese)

Davis G A. Estimating volatility and dividend yield when valuing option to invest or abandon. The Quarterly Review of Economics and Finance. 1998. 38(3): 725-754

Dixit A K and Pindyck R S. Investment under Uncertainty. New Jersey: Princeton University Press, 1994

Ge A J, Guo P and Xu H. Theory and Practice of Oil and Gas Cooperation. Beijing: Petroleum Industry Press. 2004 (in Chinese)

Hao H and Kaiser M J. Modeling China's offshore production sharing contracts using meta-analysis. Petroleum Science. 2010. 07(2): 283-288

Hull J C. Options, Futures and Other Derivative Securities, fourth ed. New Jersey: Prentice Hall, 2000

Johnston D. International Exploration Economics, Risk, and Contract Analysis. Tulsa: PennWell Books. 2003

Johnston D. International Petroleum Fiscal Systems and Production Sharing Contracts. Tulsa: PennWell Books. 1994

Lin M and Liu Z B. Stochastic simulation on the system of oil price. System Engineering. 2008. 26(2): 99-105 (in Chinese)

Luo D K and Yan N. Assessment of fiscal terms of international petroleum contracts. Petroleum Exploration and Development. 2010. 37(6): 756-762 (in Chinese)

Mudford B and Stegemeier D. Analyzing the sensitivity of production sharing contract terms using simulation. SPE Hydrocarbon

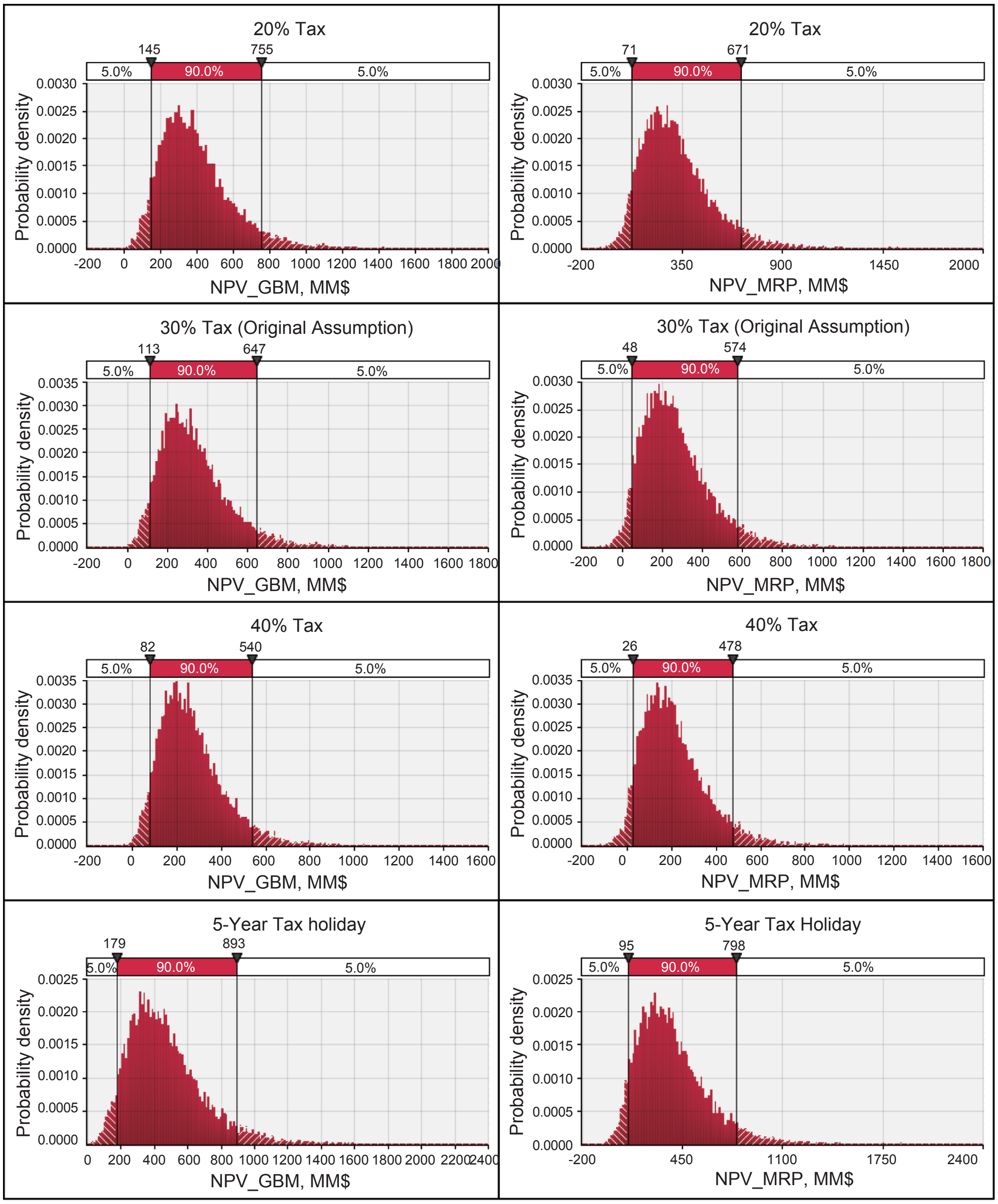

Fig. 7 Probability density graphs of NPVs at different Tax scenarios (left is under GBM and right under MRP)

Economics and Evaluation Symposium, 5-8 April 2003, Texas (SPE 82016)

Schwartz E S. The stochastic behavior of commodity prices: Implications for valuation and hedging. Journal of Finance. 1997. 52(3): 923-973

Wang Z and Li L J. Valuation of the flexibility in decision-making for revamping installations — A case from fertilizer plants. Petroleum Science. 2010. 07(3): 428-434

Wang Z, Liu M M and Zhao L. Impacts of R/T elements on contracts economics under oil price uncertainty. The 1st International Conference on Sustainable Construction & Risk Management, 12-14 June 2010a, Chongqing Jiaotong University, Chongqing

Wang Z, Zhao L and Liu M M. Impacts of PSC Elements on Contracts Economics Under Oil Price Uncertainty. 2010 International Conference on E-business and E-Government, 7-9 May 2010b, South China University of Technology, Guangzhou

Wu H Y and Wang A D. Discussion of production-sharing contract risks. Inner Mongolia Petrochemical. 2006. 7: 26-28 (in Chinese)

Zhang H L and Wang Q W. Sample size determination methods for simulation in the risk analysis of investment projects. System Engineering-Theory & Practice. 2004. 2: 14-18, 24 (in Chinese)

(Edited by Zhu Xiuqin)